EL-MOASSER

2024

Some Governorates' Examinations

Besides the examinations existing in the notebook

إهداء من سلسلة كتب المعاصر غير مخصصة للبيع

Maths



3rd PREP.

SECOND TERM

First | Algebra and Probability Exams

3rd prep.

The specification table of algebra and probability examination paper

For preparatory three (Second term) 2023/2024

Contents	Number of questions	Marks	
		The mark	The total
Accumulative basic skills	**(3)** Objective items	3	3
Equations.	**(1)** Objective item	1	10
	$\left(1\frac{1}{2}\right)$ Essay questions	9	
Algebraic fractional functions and the operations on them.	**(1)** Objective item	1	13
	(2) Essay questions	12	
Probability.	**(1)** Objective item	1	4
	$\left(\frac{1}{2}\right)$ Essay question	3	
The total	**(1)** Objective question	6	30
	(4) Essay questions	24	

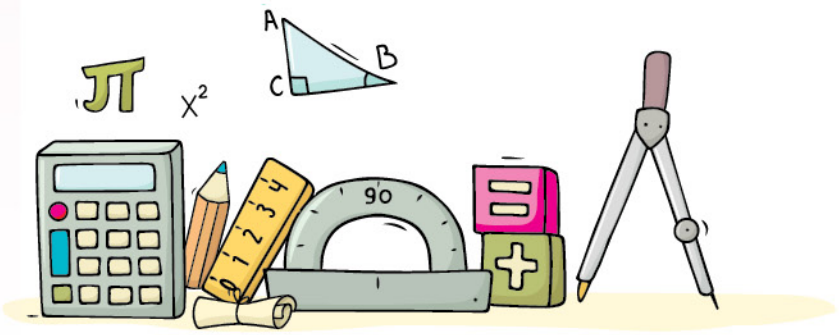
- The examination paper contains :

  Five questions ,

  First question is 6 - multiple choice items , each contains 4 alternatives , only one is true.

  This question worths 6 marks.

  Four essay questions , each contains at least two items and assign for each question 6 marks.



1 Ismailia Governorate

Answer the following questions : *(Calculator is permitted)*

1 Choose the correct answer from those given answers :

1 If $|x| = 5$, then $x =$ .....

(a) 5 (b) -5 (c) ± 5 (d) 10

2 The domain of the algebraic fraction $\frac{x-5}{7}$ equals the domain of the algebraic fraction .....

(a) $\frac{x}{x-2}$ (b) $\frac{x^2+4}{7x}$ (c) $\frac{x}{x-5}$ (d) $\frac{7}{x^2+4}$

3 If the probability to success in the preparatory certificate exam is 85 % , then the probability to fail is .....

(a) $\frac{3}{200}$ (b) $\frac{3}{20}$ (c) $\frac{17}{20}$ (d) 0.85

4 If $y^{-3} = 8$, then $y =$ .....

(a) $\frac{1}{8}$ (b) 2 (c) $\frac{1}{512}$ (d) $\frac{1}{2}$

5 If the two equations : $x + 2y = 3$, $3x + ky = 9$ have an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$, then $k =$ .....

(a) 2 (b) 3 (c) 6 (d) 9

6 If $x + 3y = 7$, then $x + 3(y + 5) =$ .....

(a) 22 (b) 12 (c) 7 (d) 35

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2 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations algebraically : $y = 2x$, $x^2 + y^2 = 20$

[b] Find n (x) in the simplest form , showing the domain where :

$$n(x) = \frac{(x-2)^3}{x^3-8} \times \frac{x^2+2x+4}{x^2-4}$$

---

3 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations algebraically : $x + 2y = 8$, $3x + y = 9$

[b] Find n (x) in the simplest form , showing the domain where :

$$n(x) = \frac{x^2+3x}{x^2+4x+3} + \frac{x-5}{x^2-4x-5}$$

---

4 [a] Using the general formula , find in $\mathbb{R}$ the solution set of the equation :

$x(x + 3) = 1$ (rounding the result to the nearest one decimal place)

[b] If $n(\mathcal{X}) = \frac{\mathcal{X}-5}{\mathcal{X}-1}$, **find :**

1 $n^{-1}(\mathcal{X})$ and identify the domain of n^{-1}

2 The value of $\mathcal{X}$ if $n(\mathcal{X}) = 3$

---

5 [a] If $n_1(\mathcal{X}) = \frac{\mathcal{X}^2+\mathcal{X}-12}{\mathcal{X}^2+5\mathcal{X}+4}$, $n_2(\mathcal{X}) = \frac{(\mathcal{X}-3)(\mathcal{X}+1)}{\mathcal{X}^2+2\mathcal{X}+1}$

, **prove that :** $n_1(\mathcal{X}) = n_2(\mathcal{X})$ for all values of $\mathcal{X}$ which belong to the common domain.

[b] If A and B are two events from the sample space of a random experiment

, $P(A) = 0.7$, $P(B) = 0.4$, $P(A \cup B) = 0.8$, **find :**

1 $P(\grave{B})$ 2 $P(A \cap B)$ 3 $P(A - B)$

2 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 If A and B are two mutually exclusive events of a random experiment , $P(A) = 0.2$, $P(A \cup B) = 0.7$, then $P(B) =$ .....

(a) zero (b) 1 (c) 0.5 (d) 0.9

2 The set of zeroes of the function f : $f(\mathcal{X}) = (\mathcal{X}-1)^2(\mathcal{X}+2)$ is .....

(a) $\{1, 2\}$ (b) $\{1, -2\}$ (c) $\{-1, 2\}$ (d) $\{-1, -2, 1\}$

3 The two straight lines : $\mathcal{X} + 5y = 1$, $\mathcal{X} + 5y - 8 = 0$ are .....

(a) coincident. (b) parallel.

(c) perpendicular. (d) intersecting and non perpendicular.

4 If $\sqrt{16+9} = 4 + \mathcal{X}$, then $\mathcal{X} =$ .....

(a) 3 (b) 4 (c) 5 (d) 1

5 If $\mathcal{X}$ is a negative number , then the greatest number of the following is .....

(a) $5\mathcal{X}$ (b) $5 + \mathcal{X}$ (c) $5 - \mathcal{X}$ (d) $\frac{5}{\mathcal{X}}$

6 $\frac{1}{\mathcal{X}} + \frac{1}{y} + \frac{1}{\mathcal{X}y} = \frac{.....}{\mathcal{X}y}$ (where $\mathcal{X} \neq$ zero , $y \neq$ zero)

(a) 2 (b) 3 (c) $\mathcal{X} + y$ (d) $\mathcal{X} + y + 1$

---

2 [a] **Find algebraically the solution set of the two equations in** $\mathbb{R} \times \mathbb{R}$ **:** $y = \mathcal{X} + 4$, $\mathcal{X} + y = 4$

[b] **Find n** $(\mathcal{X})$ **in the simplest form , showing the domain of n :** $n(\mathcal{X}) = \frac{\mathcal{X}}{\mathcal{X}-4} - \frac{\mathcal{X}+4}{\mathcal{X}^2-16}$

3 **[a] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$x^2 = 2(x + 6)$ (rounding the results to two decimal places)

[b] Find n (x) in the simplest form , showing the domain of n :

$$n(x) = \frac{x^3 - 8}{x^2 + x - 6} \times \frac{x + 3}{x^2 + 2x + 4}$$

---

4 **[a]** If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$ **, then prove that :** $n_1 = n_2$

[b] Find algebraically the solution set of the two equations in $\mathbb{R} \times \mathbb{R}$:

$x = y + 1$, $(x - y)^2 + y = 3$

---

5 **[a]** If $n(x) = \frac{x - 2}{x^2 - 5x + 6}$ **, then find :** $n^{-1}(x)$, showing the domain of n^{-1}

[b] A box contains 15 identical cards numbered from 1 to 15 , a card is drawn randomly.

Find the probability that the number on the card is :

1 divisible by 3 2 an odd number and divisible by 3

3 Beni Suef Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from those given :**

1 $\mathbb{Q} \cup \grave{\mathbb{Q}} =$ .....

(a) $\mathbb{R}$ (b) $\mathbb{Q}$ (c) $\mathbb{Z}$ (d) $\varnothing$

2 If there are an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$ of the two equations :

$x + 3y = 2$, $2x + my = 4$, then m = ...

(a) 2 (b) 3 (c) 4 (d) 6

3 $(-1)^{15}$ ..... $(-1)^8$

(a) $\geq$ (b) $>$ (c) $<$ (d) $=$

4 If A and B are two mutually exclusive events from the sample space of a random experiment , then P (A – B) = ...

(a) P (A) (b) P (B) (c) P $(\grave{A})$ (d) P $(\grave{B})$

5 If $\sqrt{16 + 9} = 4 + a$, then a = ...

(a) zero (b) 1 (c) 2 (d) 3

6 The set of zeroes of the function f : $f(x) = \frac{x + 1}{x^2 - 1}$ is .....

(a) $\{1 , -1\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\varnothing$

2 **[a] Find n ($\mathcal{X}$) in the simplest form , showing the domain of the function n where :**

$n(\mathcal{X}) = \frac{\mathcal{X}^2 + 2\mathcal{X}}{\mathcal{X}^2 - 4} + \frac{\mathcal{X} + 3}{\mathcal{X}^2 + \mathcal{X} - 6}$

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $2\mathcal{X} + y = 1$ and $\mathcal{X} + 2y = 5$

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3 **[a]** If the domain of the function $f : f(\mathcal{X}) = \frac{\mathcal{X} + 1}{\mathcal{X}^2 - a\mathcal{X} + 25}$ is $\mathbb{R} - \{5\}$ **, then find :** the value of a

[b] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :

$3\mathcal{X}^2 = 5\mathcal{X} - 1$ (rounding the result to two decimal places)

---

4 **[a] Find n ($\mathcal{X}$) in the simplest form , showing the domain of the function n where :**

$n(\mathcal{X}) = \frac{2\mathcal{X} - 4}{\mathcal{X}^3 - 8} \div \frac{\mathcal{X} + 2}{\mathcal{X}^2 + 2\mathcal{X} + 4}$, then find n (0) if possible.

[b] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $\mathcal{X} - y = 0$ and $\mathcal{X}^2 + \mathcal{X}y + y^2 = 27$

---

5 **[a]** If $n_1(\mathcal{X}) = \frac{\mathcal{X} + 5}{\mathcal{X}^2 + 4\mathcal{X} - 5}$ and $n_2(\mathcal{X}) = \frac{2\mathcal{X} - 1}{2\mathcal{X}^2 - 3\mathcal{X} + 1}$

, then prove that : $n_1(\mathcal{X}) = n_2(\mathcal{X})$ for all values of $\mathcal{X}$ which belong to the common domain and find this domain.

[b] If A and B are two events from the sample space of a random experiment , $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.8$ **, find :**

1 $P(A \cap B)$ 2 The probability of non-occurrence of the event A

4 El-Menia Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 If $\mathcal{X}y = 5$, $\mathcal{X}y^2 = 20$, then y = ...

(a) 2 (b) – 2 (c) ± 2 (d) 4

2 If $\mathcal{X}^3 + 8 = (\mathcal{X} + 2)(\mathcal{X}^2 + k\mathcal{X} + 4)$, then k = ...

(a) – 2 (b) 2 (c) – 4 (d) 4

3 If $y^2 - k = (y - 3)(y + 3)$, then k = ...

(a) 9 (b) – 6 (c) – 9 (d) 6

4 The number of solutions of the two equations : $\mathcal{X} + y = 2$ and $4 - 2\mathcal{X} = 2y$ together is .....

(a) zero (b) 1 (c) 2 (d) an infinite number.

5 The set of zeroes of the function f where $f(x) = x^2 + 9$ is .....

(a) $\mathbb{R}$ (b) $\{3, -3\}$ (c) $\mathbb{R} - \{3, -3\}$ (d) $\varnothing$

6 If $X \subset Y$, then $P(X \cap Y)$ = ...

(a) X (b) Y (c) P (X) (d) P (Y)

---

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $2y = 3 + x$, $2x = 4 - y$

[b] If the domain of n : $n(x) = \frac{x-3}{x^2 + b x + 3}$ is $\mathbb{R} - \{1, 3\}$ **, then find :** the value of b

---

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x + y = 0$ and $x^2 + y^2 = 2$

[b] If $n_1(x) = \frac{x-2}{x^2 - 4x + 4}$, $n_2(x) = \frac{1}{x-2}$

, prove that : $n_1(x) = n_2(x)$ for all the values of x belonging to the common domain.

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4 **[a] Find in $\mathbb{R}$ the solution set of the equation :** $x^2 + 1 = 3x$ rounding the results to two decimal places.

[b] Find n (x) in the simplest form , showing the domain : $n(x) = \frac{x+2}{x^2 - 4} + \frac{2x}{2x - 4}$

---

5 **[a]** If A and B are two mutually exclusive events , $P(B) = 0.3$, $P(A \cup B) = 0.8$ **, then find :**

1 $P(A \cap B)$ 2 $P(A)$

[b] Find n (x) in the simplest form , showing the domain of n :

$n(x) = \frac{x^2 + 2x}{x^3 + 8} \div \frac{2x}{x^2 - 2x + 4}$

5 Assiut Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer :**

1 If the solution set of the two equations : $x + 2y = 5$, $2x + ky = 3$ in $\mathbb{R} \times \mathbb{R}$ equals $\varnothing$, then k = ...

(a) 2 (b) – 2 (c) 4 (d) – 4

2 If $0.0043 = 4.3 \times 10^x$, then x = ...

(a) – 4 (b) – 3 (c) 4 (d) 3

3 If $n_1 = n_2$, $n_1(x) = \frac{5x}{5x^2 + 20}$, then $n_2(x)$ = ...

(a) $\frac{x}{x^2 - 4}$ (b) $\frac{x}{x^2 + 4}$ (c) $\frac{x^2 + 4}{x}$ (d) $\frac{x^2 - 4}{x}$

4 If $A \subset S$ of a random experiment and $P(A) + P(\acute{A}) = 2k$, then k = ...

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

5 If $\sqrt{x+9} = 5$, then $\sqrt{x}$ = ...

(a) 4 (b) 16 (c) 25 (d) – 4

6 If b + | – 9 | = zero , then b = ...

(a) 9 (b) – 9 (c) ± 9 (d) zero

---

2 **[a] Find in $\mathbb{R}$ the solution set of the equation : $x(x+8)+9=0$ by using the general formula , rounding the result to one decimal place.**

[b] Find n (x) in the simplest form , showing the domain of n , where :

$$n(x) = \frac{x^2+4x+3}{x^3-27} \div \frac{x+3}{x^2+3x+9}$$

---

3 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $x - y = 1$, $x^2 + y^2 = 25$**

[b] If $n(x) = \frac{x^2-x}{x^2-1} + \frac{x+5}{x^2+6x+5}$, find n (x) in the simplest form , showing the domain of n

---

4 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations algebraically :**

$2x - y = 3$, $x + 2y = 4$

[b] If $n(x) = \frac{x^2-2x}{x^2-3x+2}$ **, find if possible :**

1 $n^{-1}(x)$ in the simplest form , showing the domain of n^{-1}

2 $n^{-1}(2)$

---

5 **[a] Find the common domain for the two functions n_1 , n_2 where :**

$$n_1(x) = \frac{x-4}{x^2-5x+6} \quad , \quad n_2(x) = \frac{2x}{x^3-9x}$$

[b] If A and B are two events from the sample space of a random experiment , $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ **, find P (A ∪ B) in each of the following cases :**

1 $P(A \cap B) = \frac{1}{8}$ 2 A and B are mutually exclusive events.

6 Qena Governorate

محافظة قنا

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from those given :**

1 The solution set of the two equations : $x = 3$, $y = 4$ in $\mathbb{R} \times \mathbb{R}$ is .....

(a) $\{(3, 4)\}$ (b) $\{(4, 3)\}$ (c) $\mathbb{R}$ (d) $\varnothing$

2 If a – b = 3 , then $a^2 - 2ab + b^2$ = ...

(a) 3 (b) – 3 (c) 9 (d) – 9

3 If z $(f) = \{2\}$, $f(x) = x^3 - m$, then m = ...

(a) $\sqrt[3]{2}$ (b) 2 (c) 4 (d) 8

4 $\sqrt{x^2}$ = ...

(a) $\pm x$ (b) $|x|$ (c) $-x$ (d) x

5 If A and B are two mutually exclusive events in the sample space of a random experiment , then P $(A \cap \grave{B})$ = ...

(a) zero (b) 1 (c) 0.5 (d) $\varnothing$

6 If $(x-2)^{zero} = 1$, then $x \neq$ .....

(a) zero (b) 1 (c) 2 (d) – 2

---

2 **[a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :** $x - y = 2$, $x^2 + y^2 = 20$

[b] Find n (x) in its simplest form , showing the domain where :

$n(x) = \frac{x+1}{x^2-x-2} \times \frac{x^2+3x-10}{3x^2+16x+5}$, then find n (– 1) if possible.

---

3 **[a] Using the general formula , find in $\mathbb{R}$ the solution set of the equation :**

$x^2 - 4x + 1 =$ zero (approximate the result to the nearest two decimals)

[b] Find n (x) in its simplest form , showing the domain of n :

$n(x) = \frac{x^2+2x+4}{x^3-8} - \frac{9-x^2}{x^2+x-6}$

---

4 **[a]** Two acute angles in a right-angled triangle , the difference between their measures is 50° , find the measure of each one.

[b] If $n_1(x) = \frac{x^2}{x^3-x^2}$, $n_2(x) = \frac{x^3+x^2+x}{x^4-x}$, **prove that :** $n_1 = n_2$

---

5 **[a]** If the domain of the function n : $n(x) = \frac{(x-2)(x+1)}{x^2-k}$ is $\mathbb{R} - \{1, -1\}$, **find :**

1 The value of k

2 $n^{-1}(x)$ in the simplest form , show the domain of n^{-1}

[b] If A and B are two events in the sample space of a random experiment , and P (A) = 0.3 , P (B) = 0.6 , $P(A \cup B) = 0.7$, **find :**

1 $P(A \cap B)$ 2 P (A – B) 3 $P(\grave{A})$

7 Luxor Governorate



Answer the following questions :

1 Choose the correct answer :

1 If $X^2 + k - 9 = (X - 3)(X + 3)$, then k = ...

(a) 3 (b) – 3 (c) zero (d) 9

2 Quarter of the number 2^{20} = ...

(a) 2^5 (b) 2^{18} (c) 2^{19} (d) 2^{10}

3 If the age of Ahmed now is $X + 2$ years , then his age 5 years ago is ..... years.

(a) $X - 5$ (b) $X + 3$ (c) $5X$ (d) $X - 3$

4 The number of solutions of the two equations : $X + y = 4$, $3X + 3y = 7$ in $\mathbb{R} \times \mathbb{R}$ is .....

(a) zero (b) 1 (c) 2 (d) an infinite number.

5 The set of zeroes of the function f : $f(X) = 4$ is .....

(a) $\{4\}$ (b) $\{-4\}$ (c) $\varnothing$ (d) $\{4, -4\}$

6 If A and B are two events of the sample space of a random experiment and $A \subset B$, then $P(A \cup B)$ = ...

(a) P (A) (b) P (B) (c) zero (d) $P(A \cap B)$

---

2 [a] Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of the two equations : $2X - y = 3$, $X + 2y = 4$

[b] Find n (X) in the simplest form , showing the domain :

$n(X) = \dfrac{X^2 + 2X}{X^3 - 27} \div \dfrac{X + 2}{X^2 + 3X + 9}$, **then find :** n (2) , n (– 2) if possible.

---

3 [a] Find in $\mathbb{R}$ the solution set of the equation : $X^2 - 2X = 6$ (given that $\sqrt{7} = 2.65$)

[b] If $n(X) = \dfrac{X^2 - 4}{X^2 - 5X + 6}$, then :

1 Put n (X) in the simplest form. 2 Write the domain.

3 Find the set of zeroes of n

---

4 [a] Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations : $y - X = 2$, $X^2 + Xy - 4 = 0$

[b] Find n (X) in the simplest form , showing the domain : $n(X) = \dfrac{X^2 + 2X + 4}{X^3 - 8} - \dfrac{9 - X^2}{X^2 + X - 6}$

---

5 [a] Two acute angles in a right-angled triangle , the difference between their measures is 50°
Find the measure of each angle.

[b] If A and B are two events of the sample space of a random experiment and $P(A) = 0.5$, $P(B) = x$, $P(A \cup B) = 0.8$, **then find x if :**

1 A and B are mutually exclusive. 2 $P(A \cap B) = 0.1$

8 New Valley Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 If A and B are two events of the sample space of a random experiment where $B \subset A$, then $P(A \cup B) =$ .....

(a) $P(B)$ (b) $P(A)$ (c) $P(A \cap B)$ (d) zero

2 The set of zeroes of the function f : $f(x) = -7x$ is .....

(a) $\{-7\}$ (b) $\{7\}$ (c) $\{0\}$ (d) $\varnothing$

3 The solution set of the inequality : $4 - x < 0$ in $\mathbb{R}$ is .....

(a) $]-4, \infty[$ (b) $]4, \infty[$ (c) $]-\infty, -4[$ (d) $]-\infty, 4[$

4 The number of solutions of the equation : $x = 1$ in $\mathbb{R} \times \mathbb{R}$ is .....

(a) 0 (b) 1 (c) 2 (d) infinite.

5 The domain of the function f : $f(x) = x^2 - 1$ is .....

(a) $\{1\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{1, -1\}$ (d) $\mathbb{R}$

6 If $x^2 = 7x + y$, $y^2 = x + 7y$, $x \neq y$, then $x^2 + y^2 =$ .....

(a) 8 (b) 16 (c) 48 (d) 64

---

2 [a] Find graphically in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :

$2x - y = 5$, $x + y = 4$, then find the surface area of the triangle bounded by the two lines (representing the equations) and the y-axis.

[b] Find $n(x)$ in the simplest form , showing the domain where :

$$n(x) = \frac{x + 3}{x^2 + 2x + 4} \times \frac{x^3 - 8}{x^2 + x - 6}$$

---

3 [a] If $n(x) = \frac{x^2 - 3x}{x^2 - 4x + 3}$ **, then find :**

1 $n^{-1}(x)$ in the simplest form , showing the domain.

2 The value of x if $n^{-1}(x) = 2$

[b] Find the solution set of the two equations : $y - x = 2$, $xy + y^2 = 4$ in $\mathbb{R} \times \mathbb{R}$

4 **[a] Using the general formula , find in $\mathbb{R}$ the solution set of the equation :** $2x^2 = 1 + 5x$ (approximating the result to the nearest one decimal place)

[b] Find n (x) in the simplest form , showing the domain where : $n(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x + 5}{x^2 + 6x + 5}$

---

5 **[a]** If $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, **prove that :** $n_1 = n_2$

[b] If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, $P(A \cup B) = \frac{17}{24}$ where A and B are two events of the sample space of a random experiment , **find :**

1 $P(A \cap B)$ 2 $P(A - B)$

9 South Sinai Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1 The set of zeroes of the function f where $f(x) = x^2 + 4$ in $\mathbb{R}$ is .....

(a) $\{2\}$ (b) $\{2 , -2\}$ (c) $\mathbb{R}$ (d) $\varnothing$

2 The number of solutions of the two equations : $x + y = 2$ and $y + x = 3$ together is .....

(a) zero (b) 1 (c) $\varnothing$ (d) an infinite number.

3 If $P(A) = \frac{1}{4}$, then $P(\acute{A}) =$ .....

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

4 The domain of the function n : $n(x) = \frac{x}{x - 1}$ is .....

(a) $\mathbb{R} - \{0\}$ (b) $\mathbb{R} - \{1\}$ (c) $\mathbb{R} - \{0 , 1\}$ (d) $\mathbb{R}$

5 If $x^3 = 64$, then $\sqrt{x} =$ .....

(a) 2 (b) ± 2 (c) 4 (d) ± 8

6 If the sum of two positive numbers is 7 and their product is 12 , then the two numbers are .....

(a) 2 , 5 (b) 2 , 6 (c) 3 , 4 (d) 1 , 6

---

2 **[a] Find n (x) in the simplest form , showing the domain of the function where :**

$n(x) = \frac{x - 3}{x^2 - 7x + 12} - \frac{4}{x^2 - 4x}$

[b] By using the general formula , find in $\mathbb{R}$ the solution set of the equation :

$2x^2 - 5x + 1 = 0$ (approximating to the nearest two decimal places)

3 **[a] Prove that :** $n_1 = n_2$ where $n_1(\mathcal{X}) = \frac{2\mathcal{X}}{2\mathcal{X}+4}$ and $n_2(\mathcal{X}) = \frac{\mathcal{X}^2+2\mathcal{X}}{\mathcal{X}^2+4\mathcal{X}+4}$

[b] Find in $\mathbb{R}\times\mathbb{R}$ the solution set of the two equations : $\mathcal{X}-y=0$ and $\mathcal{X}^2+\mathcal{X}y+y^2=27$

---

4 **[a] Solve the following two equations in $\mathbb{R}\times\mathbb{R}$:** $2\mathcal{X}-y=3$, $\mathcal{X}+2y=4$

[b] Simplify to the simplest form , showing the domain : $n(\mathcal{X}) = \frac{\mathcal{X}^3-8}{\mathcal{X}^2+\mathcal{X}-6} \times \frac{\mathcal{X}+3}{\mathcal{X}^2+2\mathcal{X}+4}$

---

5 **[a]** If $n(\mathcal{X}) = \frac{\mathcal{X}^2-2\mathcal{X}}{\mathcal{X}^2-3\mathcal{X}+2}$ **, find :**

1 $n^{-1}(\mathcal{X})$ in the simplest form , showing the domain of n^{-1}

2 The value of $\mathcal{X}$ if $n^{-1}(\mathcal{X}) = 3$

[b] If A and B are two events from the sample space of a random experiment and $P(A)=0.3$, $P(B)=0.6$, $P(A\cap B)=0.2$ **, find :**

1 $P(A\cup B)$ 2 $P(\grave{B})$

10 Red Sea Governorate

Answer the following questions :

1 **Choose the correct answer from the given answers :**

1 If the two equations : $\mathcal{X}+4y=7$ and $3\mathcal{X}+ky=21$ have an infinite number of solutions in $\mathbb{R}\times\mathbb{R}$, then k = ...

(a) 4 (b) 7 (c) 12 (d) 21

2 The domain of the function f : $f(\mathcal{X}) = \mathcal{X}-3$ is .....

(a) $\mathbb{R}-\{3\}$ (b) $\mathbb{R}-\{-3\}$ (c) $\{zero\}$ (d) $\mathbb{R}$

3 The additive inverse of $|-3|$ is .....

(a) 3 (b) -3 (c) ± 3 (d) $\frac{1}{3}$

4 The set of zeroes of the function f : $f(\mathcal{X}) = \frac{\mathcal{X}^2-9}{\mathcal{X}-2}$ is .....

(a) $\{2\}$ (b) $\mathbb{R}-\{2\}$ (c) $\{3,-3\}$ (d) $\{3,2,-3\}$

5 If A and B are two mutually exclusive events in a random experiment , then $P(A-B)$ = ...

(a) $P(A)$ (b) $P(B)$ (c) $P(\grave{A})$ (d) $\varnothing$

6 If $f(\mathcal{X}) = 3$, then $f(-1)$ = ...

(a) 1 (b) 3 (c) -1 (d) -3

---

2 **[a] Find the solution set of the two equations :** $\mathcal{X}+y=4$, $2\mathcal{X}-y=2$ in $\mathbb{R}\times\mathbb{R}$

[b] If $n_1(\mathcal{X}) = \frac{\mathcal{X}^2+3\mathcal{X}+2}{\mathcal{X}^2-4}$, $n_2(\mathcal{X}) = \frac{\mathcal{X}^2-1}{\mathcal{X}^2-3\mathcal{X}+2}$

, prove that : $n_1(\mathcal{X}) = n_2(\mathcal{X})$ for all values of $\mathcal{X}$ which belong to the common domain , then find the common domain.

3 **[a] Find n $(\mathcal{X})$ in its simplest form , showing the domain where :**

$$n(\mathcal{X}) = \frac{\mathcal{X}^3 - 8}{\mathcal{X}^2 - 3\mathcal{X} + 2} \times \frac{\mathcal{X} + 1}{\mathcal{X}^2 + 2\mathcal{X} + 4}$$

[b] Find the solution set of the equation : $\mathcal{X}^2 - 2\mathcal{X} - 6 = 0$ in $\mathbb{R}$ using the general rule , rounding the results to the nearest one decimal place.

---

4 **[a] Solve the two equations :** $\mathcal{X} - y = 1$, $\mathcal{X}^2 - y^2 = 25$ in $\mathbb{R} \times \mathbb{R}$

[b] Find n $(\mathcal{X})$ in its simplest form , showing the domain where : $n(\mathcal{X}) = \frac{\mathcal{X}^2 + \mathcal{X}}{\mathcal{X}^2 - 1} - \frac{\mathcal{X} + 5}{\mathcal{X}^2 + 4\mathcal{X} - 5}$

---

5 **[a]** If $n(\mathcal{X}) = \frac{2\mathcal{X} - 4}{\mathcal{X}^2 - 4}$, **find in the simplest form :** $n^{-1}(\mathcal{X})$, showing the domain of n^{-1}

[b] If A and B are two events from the sample space of a random experiment and $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, **find :** 1 $P(A \cup B)$ 2 $P(A - B)$ 3 $P(\grave{A})$

Second Geometry Exams

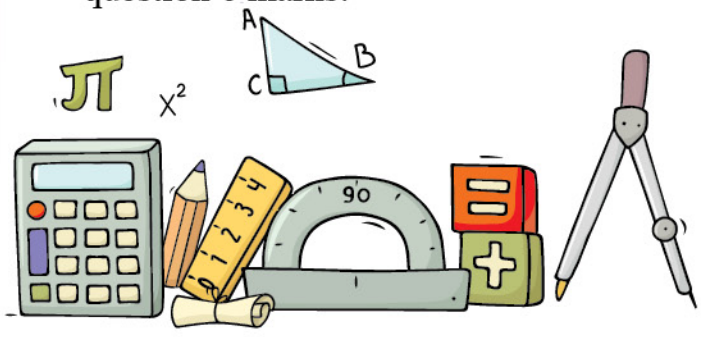
3rd prep.

The specification table of geometry examination paper

For preparatory three (Second term) 2023/2024

Contents	Number of questions	Marks	
		The mark	The total
Accumulative basic skills	(3) Objective items	3	3
* **Basic definitions and concepts.** * **Position of a point , a straight line , a circle with respect to a circle.** * **Identifying the circle.** * **The relation between the chords of a circle and its centre.**	(1) Objective item	1	7
	(1) Essay question	6	
* **Central angles and measuring arcs.** * **The relation between the inscribed and central angles.** * **Inscribed angles subtended by the same arc.**	(1) Objective item	1	7
	(1) Essay question	6	
* **The cyclic quadrilateral** * **Properties of the cyclic quadrilateral.**	(1) Objective item	1	7
	(1) Essay question	6	
* **The relation between the tangents of a circle.** * **Angles of tangency.**	(1) Essay question	6	6
The total	(1) Objective question	6	30
	(4) Essay questions	24	

- The examination paper contains :
  Five questions ,
  First question is 6 - multiple choice items , each contains 4 alternatives , only one is true.
  This question worths 6 marks.
  Four essay questions , each contains at least two items and assign for each question 6 marks.



1 Suez Governorate

Answer the following questions : *(Calculator is permitted)*

1 Choose the correct answer from those given :

1 M and N are two circles , their radii lengths are 5 cm. and 3 cm. respectively , MN = 8 cm. , then the two circles are .....

(a) distant. (b) touching externally.

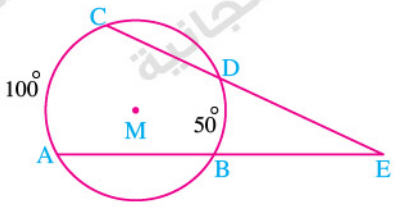
(c) concentric. (d) touching internally.

2 The sum of measures of two supplementary angles equals .....

(a) 360° (b) 270° (c) 180° (d) 90°

3 **In the opposite figure :**

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m $(\overset{\frown}{AC}) = 100°$, m $(\overset{\frown}{DB}) = 50°$, then m $(\angle E)$ = ...



(a) 25° (b) 75°

(c) 50° (d) 30°

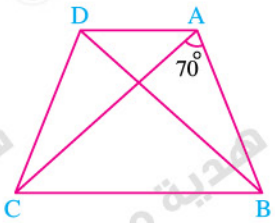
4 If the measures of two angles in a triangle are 40° and 100° , then the triangle is .....

(a) a scalene triangle. (b) an isosceles triangle.

(c) an equilateral triangle. (d) a right-angled triangle.

5 **In the opposite figure :**

ABCD is a cyclic quadrilateral , m $(\angle BAC) = 70°$, then m $(\angle BDC)$ = ...



(a) 35° (b) 70°

(c) 140° (d) 80°

6 If $\overrightarrow{BD}$ bisects $\angle ABC$, m $(\angle ABC) = 60°$, then m $(\angle ABD)$ = ...

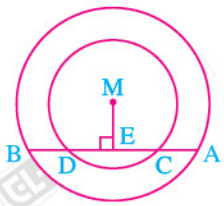
(a) 120° (b) 60° (c) 40° (d) 30°

---

2 [a] In the opposite figure :

Two concentric circles with centre M , $\overline{AB}$ is a chord in the greater circle and intersects the smaller at C , D , $\overline{ME} \perp \overline{AB}$

Prove that : AC = DB



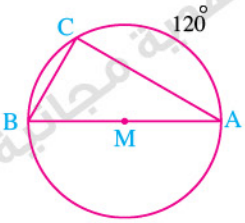
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m $(\overset{\frown}{AC}) = 120°$

Find : 1 m (∠ C)

2 m (∠ A)



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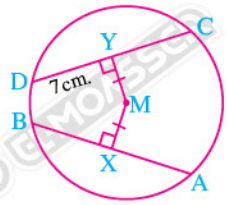
3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

, MX = MY , YD = 7 cm.

Find : the length of $\overline{AB}$

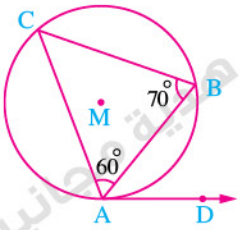


[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M at A

, m (∠ ABC) = 70° , m (∠ BAC) = 60°

Find : m (∠ BAD)



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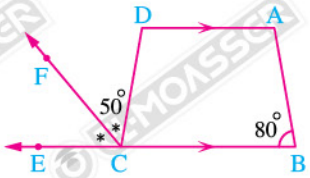
4 **[a]** State two cases of a cyclic quadrilateral.

[b] In the opposite figure :

$\overline{AD}$ // $\overrightarrow{BC}$, $\overrightarrow{CF}$ bisects ∠ DCE

, m (∠ DCF) = 50° , m (∠ B) = 80°

Prove that : ABCD is a cyclic quadrilateral.



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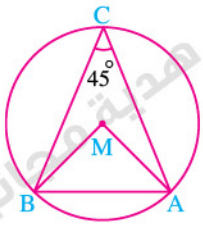
5 **[a] In the opposite figure :**

Δ ABC is inscribed in the circle M

, m (∠ ACB) = 45°

Find : 1 m (∠ AMB)

2 m (∠ MAB)



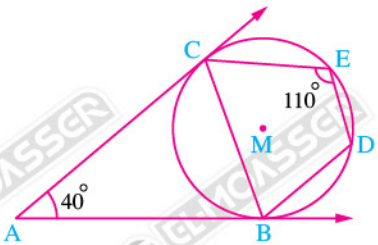
[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to

the circle at B , C

, m (∠ E) = 110° , m (∠ A) = 40°

Prove that : m (∠ ABC) = m (∠ DBC)



2 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

1 The measure of the arc which equals one third of the measure of the circle equals .....

(a) 60° (b) 120° (c) 180° (d) 270°

2 In the right-angled triangle , the length of the median drawn from the right angle equals ..... the length of the hypotenuse.

(a) fourth (b) one third (c) half (d) twice

3 M and N are two touching externally circles with radii lengths 9 cm. and 5 cm. , then MN = ... cm.

(a) 5 (b) 9 (c) 4 (d) 14

4 Δ ABC is a right-angled triangle at B , AB = 6 cm. , AC = 10 cm. , then BC = ... cm.

(a) 6 (b) 8 (c) 10 (d) 4

5 If ABCD is a cyclic quadrilateral , m (∠ B) = 70° , then m (∠ D) = ...

(a) 20° (b) 70° (c) 110° (d) 90°

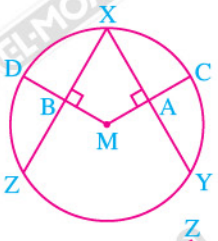
6 If a straight line intersects two parallel straight lines , then each two alternate angles are .....

(a) equal in measure. (b) supplementary. (c) complementary. (d) otherwise.

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2 [a] In the opposite figure :

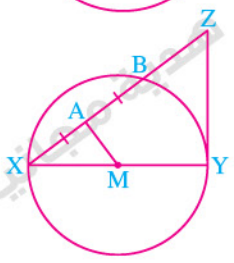
If XY = XZ , $\overrightarrow{MA} \perp \overline{XY}$, $\overrightarrow{MB} \perp \overline{XZ}$, **prove that :** AC = BD



[b] In the opposite figure :

$\overline{XY}$ is a diameter in a circle M , $\overline{YZ}$ is a tangent-segment at Y , A is the midpoint of $\overline{XB}$

Prove that : the figure MAZY is a cyclic quadrilateral.

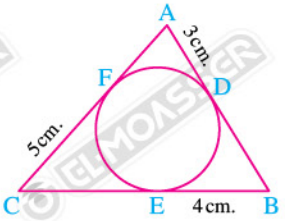


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3 [a] In the opposite figure :

A circle touches the sides of a triangle $\overline{AB}$, $\overline{BC}$, $\overline{CA}$ at D , E , F respectively , AD = 3 cm. , BE = 4 cm. , CF = 5 cm.

Find : the perimeter of Δ ABC



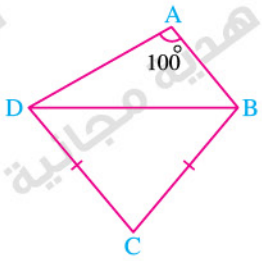
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, BC = DC

, m (∠ A) = 100°

Find : m (∠ CBD)



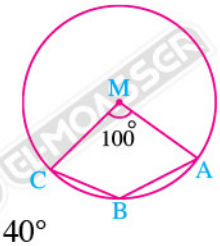
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4 [a] In the opposite figure :

M is a circle , $\overline{MA}$ and $\overline{MC}$ are two radii

, m (∠ AMC) = 100° , B $\in \overset{\frown}{AC}$

Find : m (∠ ABC)



[b] $\overline{AB}$ is a diameter in a circle , draw the two chords $\overline{AC}$ and $\overline{BC}$, m (∠ A) = 40°

Find by proof : m (∠ B)

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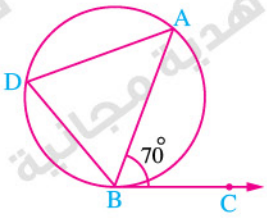
5 [a] In the opposite figure :

ABD is a triangle inscribed in a circle

, $\overrightarrow{BC}$ is a tangent to the circle at B , m (∠ ABC) = 70°

Find : 1 m (∠ D)

2 m ($\overset{\frown}{AB}$) "the minor arc"



[b] Using your geometric tools , draw $\overline{AB}$ with length 5 cm. , then draw on one figure a circle passing through the two points A and B and its diameter length is 5 cm. What are the possible solutions ?

3 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from the given answers :

1 The circumference of a circle is 20 π cm. , then its area equals ..... cm^2

(a) 10 π (b) 100 π (c) 200 π (d) 400 π

2 The measure of the interior angle of the regular pentagon equals .....

(a) 60° (b) 90° (c) 108° (d) 120°

3 Two circles M , N are touching internally , if their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

4 The measure of the inscribed angle is ..... the measure of the central angle subtended by the same arc.

(a) third (b) quarter (c) half (d) twice

5 The measure of the exterior angle at any vertex of the equilateral triangle equals .....

(a) 30° (b) 45° (c) 60° (d) 120°

6 If ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) = ...

(a) 70° (b) 90° (c) 110° (d) 130°

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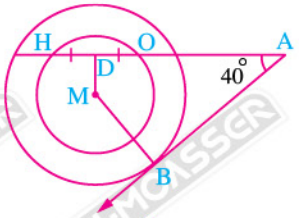
2 **[a] In the opposite figure :**

Two concentric circles at M

, D is the midpoint of $\overline{OH}$, m (∠ A) = 40°

, $\overrightarrow{AB}$ is a tangent to the big circle at B

Find with proof : m (∠ DMB)

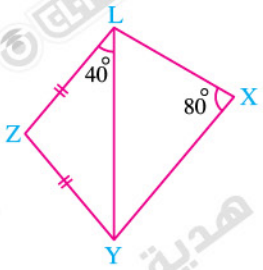


[b] In the opposite figure :

XYZL is a quadrilateral , m (∠ YLZ) = 40°

, m (∠ X) = 80° , LZ = ZY

Prove that : XYZL is a cyclic quadrilateral.



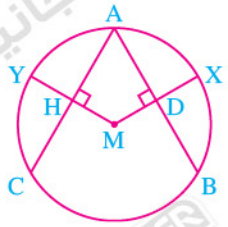
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3 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two chords in the circle M

, AB = AC , $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

Prove that : DX = HY



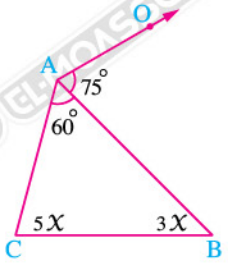
[b] In the opposite figure :

ABC is a triangle , m (∠ BAC) = 60° , m (∠ B) = 3 $\mathcal{X}^\circ$

, m (∠ C) = 5 $\mathcal{X}^\circ$, m (∠ BAO) = 75°

Find with proof : the value of $\mathcal{X}$

, then prove that : $\overrightarrow{AO}$ is a tangent to the circle which passes through the points A , B and C



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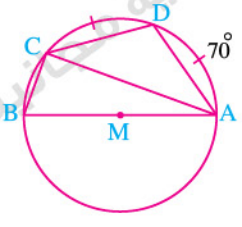
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, D is the midpoint of $\overset{\frown}{ADC}$

, m ($\overset{\frown}{AD}$) = 70°

Find with proof : m (∠ ADC) , m (∠ DCB)

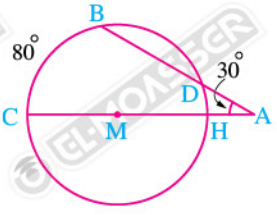


[b] In the opposite figure :

$\overline{AB} \cap \overline{AC} = \{A\}$, m (∠ A) = 30°

, m ($\overset{\frown}{BC}$) = 80°

Find with proof : m ($\overset{\frown}{HD}$)

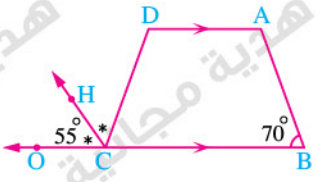


5 **[a] In the opposite figure :**

$\overline{AD} \parallel \overrightarrow{BC}$, $\overrightarrow{CH}$ bisects $\angle DCO$

, $m(\angle B) = 70°$, $m(\angle OCH) = 55°$

Prove that : ABCD is a cyclic quadrilateral.

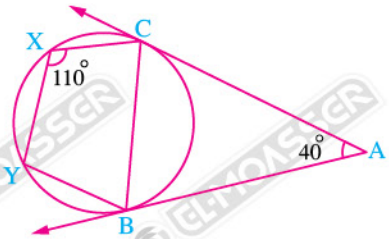


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, $m(\angle A) = 40°$, $m(\angle X) = 110°$

Prove that : $\overrightarrow{BC}$ bisects $\angle ABY$



4 Beni Suef Governorate

Answer the following questions : ***(Calculator is permitted)***

1 **Choose the correct answer from those given :**

1 ABCD is a square , then $m(\angle ACB)$ = ...

(a) 90° (b) 60° (c) 45° (d) 30°

2 If the length of $\overline{AB}$ = 6 cm. , then the radius length of the smallest circle which passes through the two points A and B equals ..... cm.

(a) 2 (b) 3 (c) 6 (d) 12

3 ΔABC is right-angled at B , D is the midpoint of $\overline{AC}$, then BD = ...

(a) AB (b) BC (c) AC (d) AD

4 ABCD is a cyclic quadrilateral , if $m(\angle ACB) = 80°$, then $m(\angle ADB)$ = ...

(a) 40° (b) 80° (c) 100° (d) 160°

5 If the ratio between the lengths of two corresponding sides of two similar polygons equals 3 : 4 , then the ratio between their perimeters equals .....

(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

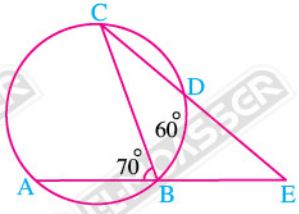
6 **In the opposite figure :**

$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, $m(\angle ABC) = 70°$

and $m(\overset{\frown}{DB}) = 60°$, then $m(\angle E)$ = ...

(a) 40° (b) 60°

(c) 80° (d) 100°

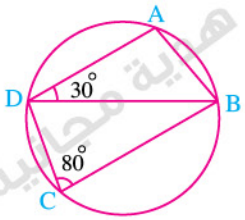


2 **[a] In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle

, m (∠ C) = 80° and m (∠ ADB) = 30°

Find : m (∠ ABD)

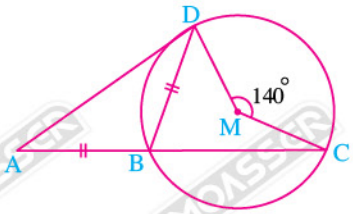


[b] In the opposite figure :

A circle with centre M , $\overleftrightarrow{CB} \cap \overleftrightarrow{DA} = \{A\}$

, m (∠ DMC) = 140° and AB = BD

Find : m (∠ BAD)



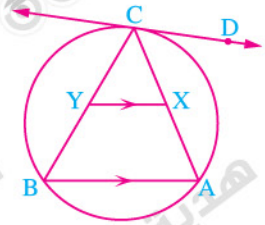
---

3 **[a] In the opposite figure :**

Δ ABC is inscribed in a circle

, $\overleftrightarrow{CD}$ is a tangent to the circle at C and $\overline{AB}$ // $\overline{XY}$

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ XYC



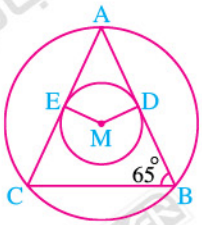
[b] In the opposite figure :

Two concentric circles M , m (∠ B) = 65°

, $\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the smaller circle at D and E

1 **Prove that :** AB = AC 2 **Find :** m (∠ BAC)



---

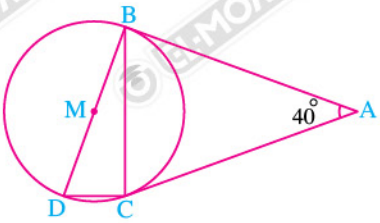
4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle M at B and C

, $\overline{BD}$ is a diameter in the circle and m (∠ BAC) = 40°

Find : m (∠ CBD)

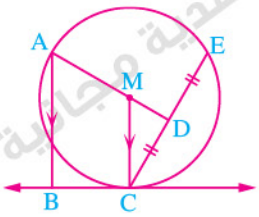


[b] In the opposite figure :

$\overleftrightarrow{BC}$ is a tangent to the circle M at C

, M ∈ $\overline{AD}$, D is the midpoint of $\overline{CE}$ and $\overline{AB}$ // $\overline{MC}$

Prove that : the figure ABCD is a cyclic quadrilateral.



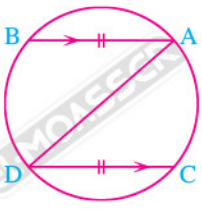
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5 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in a circle

, AB = CD and $\overline{AB}$ // $\overline{CD}$

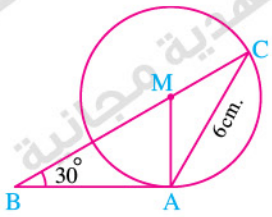
Prove that : $\overline{AD}$ is a diameter in the circle.



[b] In the opposite figure :

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

, $\overrightarrow{AB} \cap \overrightarrow{CM} = \{B\}$, m (∠ ABC) = 30° and AC = 6 cm.



Find : 1 m (∠ AMC)

2 The length of $\overline{AB}$

5 Souhag Governorate

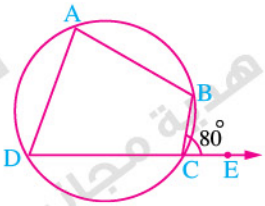


Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 In the opposite figure :

m (∠ A) = ...



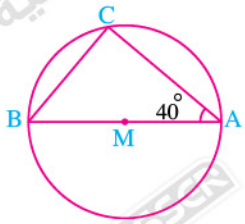
(a) 100° (b) 180°

(c) 80° (d) 360°

2 In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m (∠ CAB) = 40° , then m (∠ B) = ...



(a) 50° (b) 40°

(c) 100° (d) 70°

3 The number of circles whose centre is (7 , 4) and pass through the point (3 , 1) is .....

(a) 2 (b) 1 (c) 5 (d) infinite.

4 The number of symmetry axes of an equilateral triangle equals .....

(a) 1 (b) 2 (c) 3 (d) 4

5 A circle is of circumference 6 π cm. , then the length of its diameter equals ..... cm.

(a) 6 (b) 3 (c) 12 (d) 0

6 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is ..... cm^2

(a) 2 (b) 14 (c) 24 (d) 48

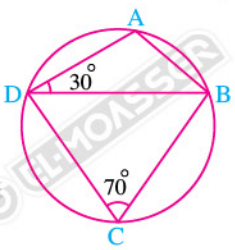
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2 [a] In the opposite figure :

m (∠ ADB) = 30°

, m (∠ C) = 70°

Find : m (∠ ABD)

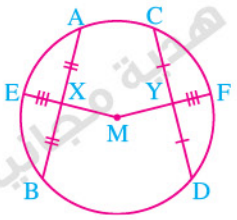


[b] In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two chords in the circle M , EX = YF

, X and Y are the midpoints of $\overline{AB}$ and $\overline{CD}$

Prove that : AB = CD



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3 **[a] In the opposite figure :**

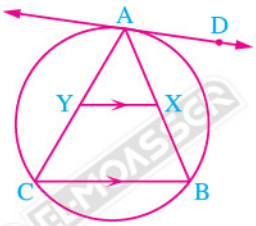
Δ ABC is inscribed in the circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

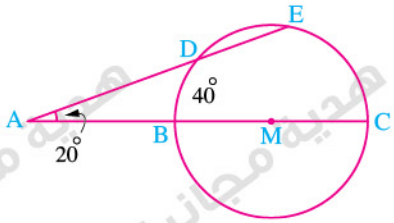


[b] In the opposite figure :

$\overline{AE} \cap \overline{AC} = \{A\}$, $\overline{BC}$ is a diameter in the circle M

, m (∠ A) = 20° , m $(\overset{\frown}{BD})$ = 40°

Find : m $(\overset{\frown}{ED})$



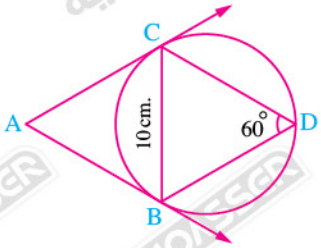
---

4 **[a] In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

, m (∠ D) = 60° , CB = 10 cm.

Find : the perimeter of Δ ABC



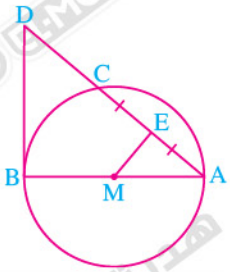
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overline{BD}$ is a tangent-segment to the circle at B

, E is the midpoint of $\overline{AC}$

Prove that : the figure MEDB is a cyclic quadrilateral.



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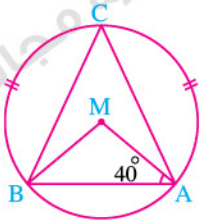
5 **[a] In the opposite figure :**

In circle M

, m (∠ MAB) = 40°

, m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BC})$

Find : m (∠ CAM)



[b] M , N are two circles , their radii lengths are 8 cm. , 6 cm. respectively. **Determine the position of each of them with respect to the other in each of the following cases :**

1 MN = 2 cm. 2 MN = 6 cm. 3 MN $\notin$]0 , ∞[

6 Aswan Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If 3 , 7 , $\mathcal{X}$ are side lengths of an isosceles triangle , then $\mathcal{X}$ = ... cm.

(a) 3 (b) 4 (c) 7 (d) 10

2 The square whose perimeter is 20 cm. , then its area is ..... cm^2

(a) 4 (b) 5 (c) 20 (d) 25

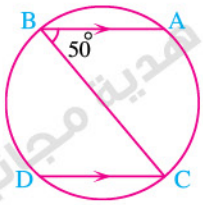
3 The parallelogram in which one of its angles is right is .....

(a) a rectangle. (b) a rhombus. (c) a square. (d) a trapezium.

4 **In the opposite figure :**

If $\overline{BA}$ // $\overline{CD}$, m (∠ B) = 50°

, then m ($\overset{\frown}{AC}$) + m ($\overset{\frown}{BD}$) = ...



(a) 50° (b) 200°

(c) 150° (d) 180°

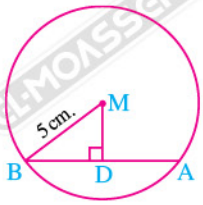
5 If $\overline{AC}$ is a diameter in a circle M , then m ($\overset{\frown}{AC}$) = ...

(a) 45° (b) 90° (c) 180° (d) 360°

6 **In the opposite figure :**

If AB = 8 cm. , MB = 5 cm. , $\overline{MD} \perp \overline{AB}$

, then MD = ... cm.



(a) 3 (b) 4

(c) 8 (d) 13

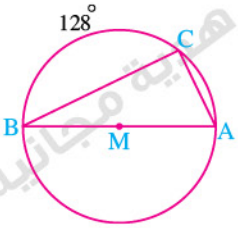
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2 [a] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m ($\overset{\frown}{BC}$) = 128°

Find : m (∠ B)



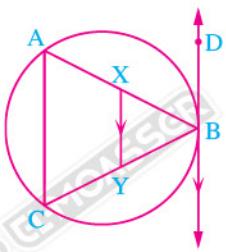
[b] In the opposite figure :

ABC is an inscribed triangle in the circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$

where $\overline{XY}$ // $\overleftrightarrow{BD}$



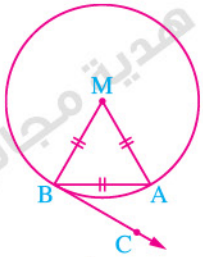
Prove that : the figure AXYC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**

MAB is an equilateral triangle

, $\overleftrightarrow{BC}$ is a tangent to the circle at B

Find : m (∠ ABC)



[b] In the opposite figure :

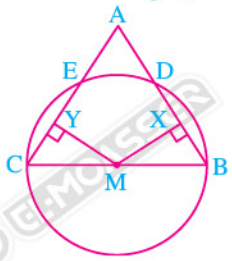
ABC is a triangle in which AB = AC

, a circle M was drawn whose diameter is $\overline{BC}$

and intersected $\overline{AB}$ at D , $\overline{AC}$ at E

, $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$

Prove that : BD = CE



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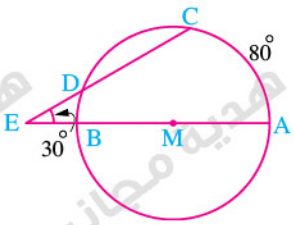
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° , m ($\overset{\frown}{AC}$) = 80°

Find : m ($\overset{\frown}{CD}$)

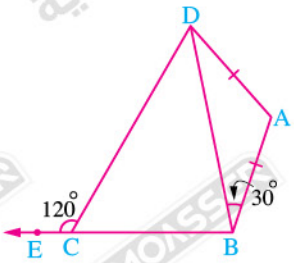


[b] In the opposite figure :

AB = AD , m (∠ ABD) = 30°

, m (∠ DCE) = 120°

Prove that : the figure ABCD is a cyclic quadrilateral.



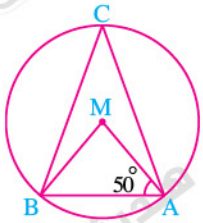
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5 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle M

, m (∠ MAB) = 50°

Find : m (∠ C)



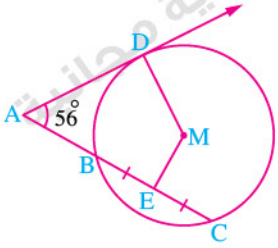
[b] In the opposite figure :

$\overleftrightarrow{AD}$ is a tangent to the circle M

, $\overline{AC}$ intersects the circle at B and C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 56°

Find : m (∠ DME)



7 New Valley Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The inscribed angle drawn in a semicircle is .....

(a) acute. (b) obtuse. (c) straight. (d) right.

2 ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ...

(a) 120° (b) 90° (c) 60° (d) 30°

3 The angle of tangency is the included angle between .....

(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

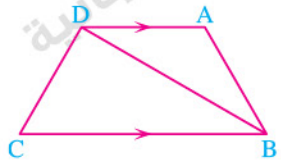
4 M and N are two intersecting circles of radii lengths 3 cm. and 5 cm. , then MN ∈ .....

(a)]8 , ∞[(b)]– ∞ , 2[(c)]0 , 2[(d)]2 , 8[

5 **In the opposite figure :**

$AD = \frac{1}{2}$ CB

, then the area of Δ ABD : the area of the figure ABCD = ...

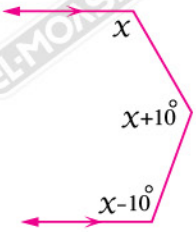


(a) 1 : 2 (b) 1 : 3

(c) 1 : 4 (d) 2 : 3

6 **In the opposite figure :**

$\tan\left(\frac{x}{2}\right)$ = ...

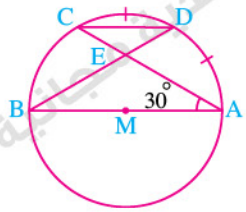


(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\sqrt{3}$ (d) 1

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2 [a] In the opposite figure :



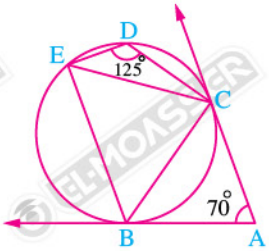
$\overline{AB}$ is a diameter in the circle M , C ∈ the circle

, m (∠ CAB) = 30° , D is the midpoint of $\overset{\frown}{AC}$

, $\overline{DB} \cap \overline{AC} = \{E\}$

1 **Find :** m ($\overset{\frown}{AD}$) 2 **Prove that :** $\overline{AB} \parallel \overline{DC}$

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C respectively

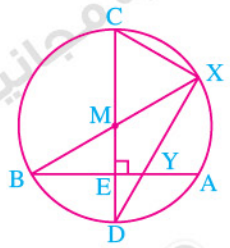
, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that : 1 CB = CE 2 $\overrightarrow{AC} \parallel \overrightarrow{BE}$

3 **[a]** State two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M , $\overline{CD}$ is a diameter perpendicular to $\overline{AB}$ and intersects it at E , $\overrightarrow{BM}$ intersects the circle at X , $\overline{XD} \cap \overline{AB} = \{Y\}$



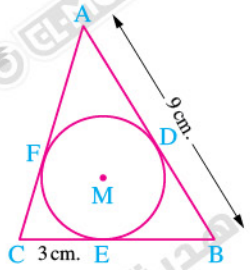
Prove that : 1 The figure XYEC is a cyclic quadrilateral.

2 m (∠ DYB) = m (∠ DBX)

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4 **[a]** **In the opposite figure :**

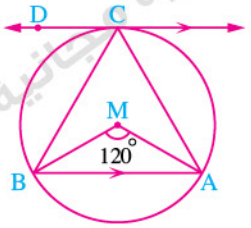
A circle is drawn touching the sides of the triangle ABC $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F respectively , AB = 9 cm. , CE = 3 cm.



Find : the perimeter of Δ ABC

[b] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C , $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

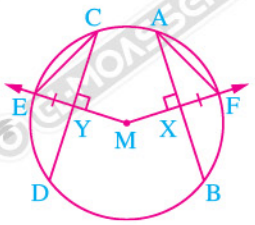


Prove that : Δ ABC is an equilateral triangle.

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5 **[a]** **In the opposite figure :**

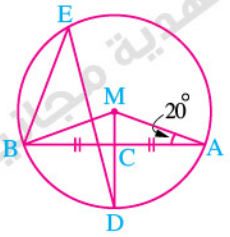
$\overline{AB}$, $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F , $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY



Prove that : 1 AB = CD 2 AF = CE

[b] **In the opposite figure :**

C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D} , m (∠ MAB) = 20°



Find : m (∠ BED) , m ($\overset{\frown}{ADB}$)

8 South Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

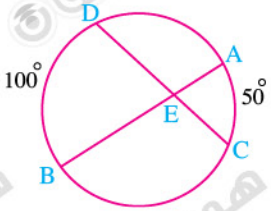
1 The measure of the arc which equals half the measure of the circle equals .....

(a) 360° (b) 180° (c) 120° (d) 90°

2 The number of common tangents of two circles touching internally equals .....

(a) zero (b) 1 (c) 2 (d) 3

3 **In the opposite figure :**



m (∠ AEC) = ...

(a) 25° (b) 50°

(c) 75° (d) 100°

4 If ABCD is a cyclic quadrilateral in which m (∠ B) = 50° , then m (∠ D) = ...

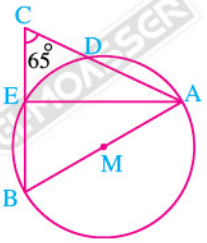
(a) 25° (b) 50° (c) 100° (d) 130°

5 If the lengths of the two diagonals of a rhombus are 3 cm. and 4 cm.

, then its area = ... cm^2

(a) 6 (b) 12 (c) 24 (d) 48

6 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

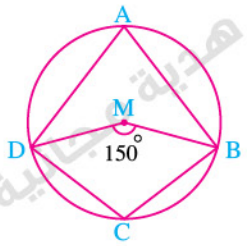
, m (∠ C) = 65° , then m (∠ CAE) = ...

(a) 65° (b) 45°

(c) 30° (d) 25°

---

2 [a] In the opposite figure :

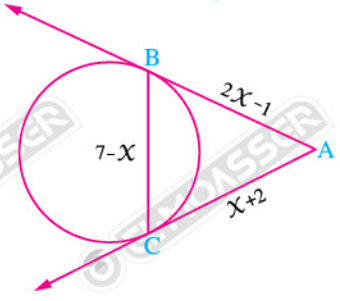


m (∠ BMD) = 150°

Find : 1 m (∠ BAD)

2 m (∠ BCD)

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

where AB = (2 X – 1) cm.

, AC = (X + 2) cm. , BC = (7 – X) cm.

Find : 1 The value of X

2 The perimeter of Δ ABC

3 **[a]** State two cases of the quadrilateral to be cyclic.

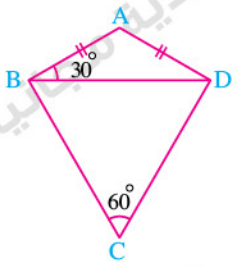
[b] **In the opposite figure :**

ABCD is a quadrilateral in which

AB = AD , m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : the figure ABCD is a cyclic quadrilateral.



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4 **[a]** **In the opposite figure :**

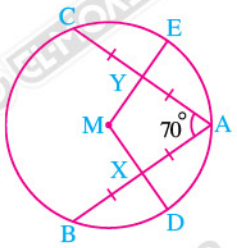
$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

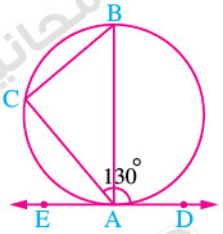


[b] **In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle at A

, m (∠ CAD) = 130°

Find with proof : m (∠ ABC)



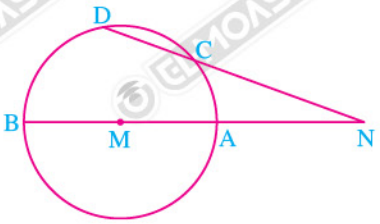
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5 **[a]** **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NC > NA

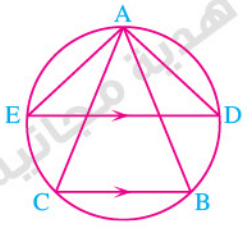


[b] **In the opposite figure :**

ABC is a triangle inscribed in the circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)



9 North Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If the surface of circle M ∩ the surface of circle N = {A} , then the two circles are .....

(a) distant. (b) touching externally.

(c) touching internally. (d) intersecting.

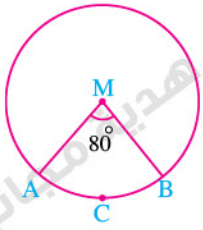
2 The sum of measures of the interior angles of the triangle equals .....

(a) 90° (b) 180° (c) 360° (d) 60°

3 **In the opposite figure :**

In the circle M , m (∠ AMB) = 80°

, then m (ACB) = ...



(a) 40° (b) 160°

(c) 80° (d) 120°

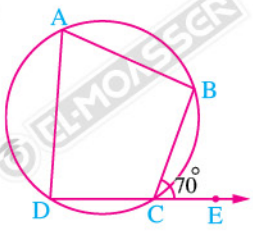
4 The area of the square whose side length is 5 cm. equals ..... cm^2

(a) 25 (b) 5 (c) 20 (d) 10

5 **In the opposite figure :**

E ∈ $\overrightarrow{DC}$, m (∠ BCE) = 70°

, then m (∠ BAD) = ...

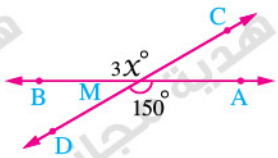


(a) 70° (b) 140°

(c) 35° (d) 120°

6 **In the opposite figure :**

$\overleftrightarrow{AB} \cap \overleftrightarrow{CD}$ = {M} , then $\mathcal{X}$ = ...



(a) 150° (b) 120°

(c) 50° (d) 360°

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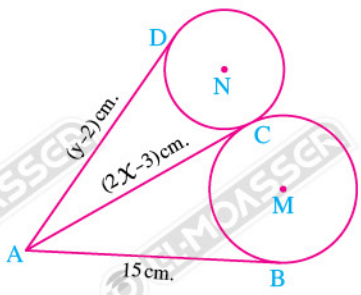
2 [a] In the opposite figure :

M , N are two circles touching externally at C

, $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

, $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the circle N

, AB = 15 cm. , AC = (2 $\mathcal{X}$ – 3) cm. , AD = (y – 2) cm.



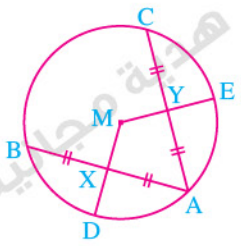
Find with proof : the value of each of $\mathcal{X}$, y

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two equal chords in the circle M

, X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively

Prove that : XD = YE



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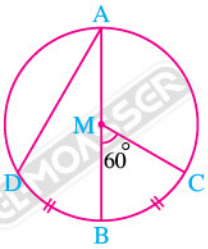
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{BD})$

, m (∠ BMC) = 60°

Find with proof : m (∠ BAD)

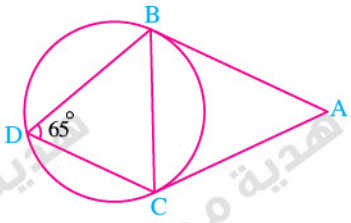


[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B , C

, m (∠ BDC) = 65°

Find with proof : m (∠ BAC)



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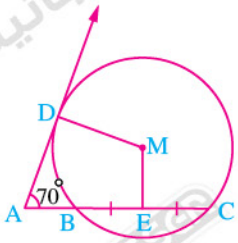
4 **[a] In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle at D

, $\overrightarrow{AC}$ is a secant of the circle at B , C

, E is the midpoint of $\overline{BC}$, m (∠ DAC) = 70°

Find with proof : m (∠ DME)



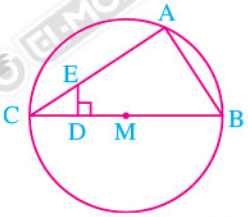
[b] In the opposite figure :

$\overline{BC}$ is a diameter in the circle M

, $\overline{ED} \perp \overline{BC}$

Prove that : 1 ABDE is a cyclic quadrilateral.

2 m (∠ CED) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$



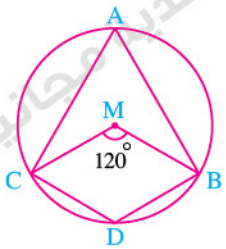
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5 **[a] In the opposite figure :**

M is a circle in which m (∠ BMC) = 120°

Find with proof :

1 m (∠ BAC) 2 m (∠ BDC)

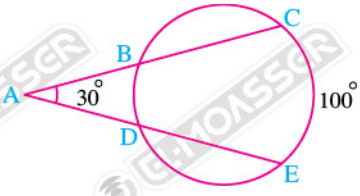


[b] In the opposite figure :

m $(\overset{\frown}{CE})$ = 100°

, m (∠ A) = 30°

Find with proof : m $(\overset{\frown}{BD})$



10 Red Sea Governorate



Answer the following questions :

1 Choose the correct answer from the given answers :

1 Two circles M and N touching internally , their radii lengths are 5 cm. and 9 cm. respectively , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

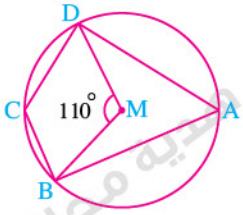
2 ABC is a triangle in which m (∠ A) = 50° and m (∠ B) = 70° , then the number of axes of symmetry of Δ ABC is .....

(a) zero (b) 1 (c) 2 (d) 3

3 **In the opposite figure :**

m (∠ BMD) = 110°

, then m (∠ C) = ...



(a) 50° (b) 70°

(c) 110° (d) 125°

4 A rhombus of side length L cm. , its perimeter = ... cm.

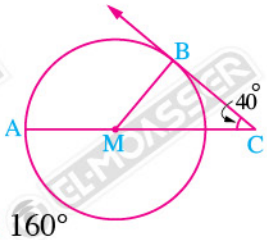
(a) L^2 (b) $2L^2$ (c) 4 L (d) $2\sqrt{2}$ L

5 **In the opposite figure :**

$\overrightarrow{CB}$ is a tangent to the circle M

, m (∠ C) = 40° , M ∈ $\overline{AC}$

, then m ($\overset{\frown}{AB}$) = ...



(a) 40° (b) 80° (c) 130° (d) 160°

6 If ∠ A and ∠ B are two supplementary angles , then m (∠ A) + m (∠ B) = ...

(a) 90° (b) 180° (c) 270° (d) 360°

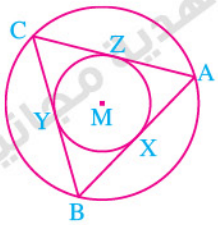
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2 [a] In the opposite figure :

Two concentric circles M , ABC is a triangle

its vertices are on the greater circle and its sides touch

the smaller circle at X , Y , Z



Prove that : ABC is an equilateral triangle.

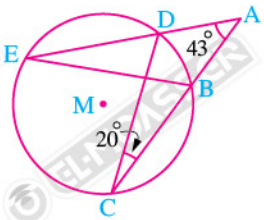
[b] In the opposite figure :

$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

, m (∠ A) = 43°

and m (∠ ACD) = 20°

Find : m (∠ E) and m ($\overset{\frown}{CE}$)



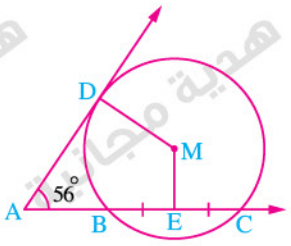
3 **[a] In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle M at D

, m (∠ A) = 56°

, E is the midpoint of $\overline{CB}$

Find : m (∠ M)

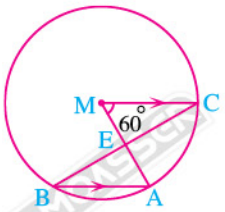


[b] In the opposite figure :

$\overline{MC}$ // $\overline{BA}$, $\overline{BC} \cap \overline{MA} = \{E\}$

, m (∠ M) = 60°

Find : m (∠ B) **and prove that :** $\overline{BC} \perp \overline{AM}$



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4 **[a] In the opposite figure :**

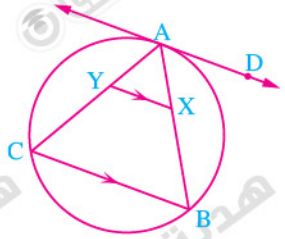
$\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$

, where $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



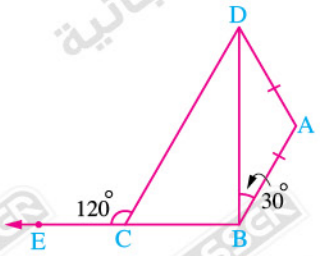
[b] In the opposite figure :

ABCD is a quadrilateral in which E ∈ $\overrightarrow{BC}$

, AB = AD , m (∠ DCE) = 120°

, m (∠ ABD) = 30°

Prove that : ABCD is a cyclic quadrilateral.



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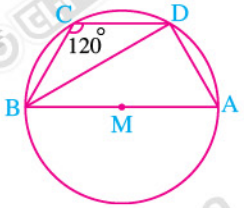
5 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

and m (∠ BCD) = 120°

Find : 1 m (∠ A)

2 m (∠ ABD)



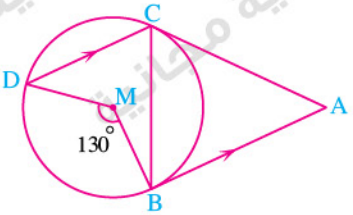
[b] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to

the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

Find : m (∠ A)



EL-MOASSER

2024

Answers



3rd PREP.

SECOND TERM

Maths

1 Ismailia

1 1 c 2 d 3 b 4 d 5 c 6 a

2

[a] $\because y = 2x$ (1)

, $x^2 + y^2 = 20$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + (2x)^2 = 20$ $\therefore x^2 + 4x^2 = 20$

$\therefore 5x^2 = 20$ $\therefore x^2 = 4$

$\therefore x = 2$ or $x = -2$

Substituting in (1) : $\therefore y = 4$ or $y = -4$

$\therefore$ The S.S. $= \{(2, 4), (-2, -4)\}$

[b] $\because n(x) = \frac{(x-2)^3}{(x-2)(x^2+2x+4)} \times \frac{x^2+2x+4}{(x+2)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

, $n(x) = \frac{x-2}{x+2}$

3

[a] $\because x + 2y = 8$ $\therefore x = 8 - 2y$ (1)

, $3x + y = 9$ (2)

Substituting from (1) in (2) :

$\therefore 3(8 - 2y) + y = 9$

$\therefore 24 - 6y + y = 9$ $\therefore 24 - 5y = 9$

$\therefore 5y = 15$ $\therefore y = 3$

Substituting in (1) : $\therefore x = 2$

$\therefore$ The S.S. $= \{(2, 3)\}$

[b] $\because n(x) = \frac{x(x+3)}{(x+1)(x+3)} + \frac{x-5}{(x-5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1, -3, 5\}$

, $n(x) = \frac{x}{x+1} + \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

4

[a] $\because x(x+3) = 1$ $\therefore x^2 + 3x - 1 = 0$

$\therefore a = 1$, $b = 3$, $c = -1$

$\therefore x = \frac{-3 \pm \sqrt{(3)^2 - 4 \times 1 \times -1}}{2 \times 1} = \frac{-3 \pm \sqrt{13}}{2}$

$\therefore x \simeq 0.3$ or $x \simeq -3.3$

$\therefore$ The S.S. $= \{0.3, -3.3\}$

[b] 1 $\because n(x) = \frac{x-5}{x-1}$ $\therefore n^{-1}(x) = \frac{x-1}{x-5}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{1, 5\}$

2 $\because n(x) = 3$ $\therefore \frac{x-5}{x-1} = 3$

$\therefore 3x - 3 = x - 5$ $\therefore 3x - x = -5 + 3$

$\therefore 2x = -2$ $\therefore x = -1$

5

[a] $\because n_1(x) = \frac{(x-3)(x+4)}{(x+4)(x+1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-4, -1\}$

, $n_1(x) = \frac{x-3}{x+1}$

, $\because n_2(x) = \frac{(x-3)(x+1)}{(x+1)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-1\}$

, $n_2(x) = \frac{x-3}{x+1}$

$\therefore n_1(x) = n_2(x)$ for all the values of $x \in \mathbb{R} - \{-4, -1\}$

[b] 1 $P(\bar{B}) = 1 - P(B) = 1 - 0.4 = 0.6$

2 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.7 + 0.4 - 0.8 = 0.3$

3 $P(A - B) = P(A) - P(A \cap B) = 0.7 - 0.3 = 0.4$

2 El-Beheira

1 1 c 2 b 3 b 4 d 5 c 6 d

2

[a] $\because y = x + 4$ (1)

, $x + y = 4$ (2)

Substituting from (1) in (2) : $\therefore x + x + 4 = 4$

$\therefore 2x = 0$ $\therefore x = 0$

Substituting in (1) : $\therefore y = 4$

$\therefore$ The S.S. $= \{(0, 4)\}$

[b] $\because n(x) = \frac{x}{x-4} - \frac{x+4}{(x-4)(x+4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{4, -4\}$

, $n(x) = \frac{x}{x-4} - \frac{1}{x-4} = \frac{x-1}{x-4}$

3

[a] $\because x^2 = 2(x+6)$

$\therefore x^2 - 2x - 12 = 0$

$\therefore a = 1$, $b = -2$, $c = -12$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times -12}}{2 \times 1} = \frac{2 \pm 2\sqrt{13}}{2} = 1 \pm \sqrt{13}$

$\therefore x \simeq 4.61$ or $x \simeq -2.61$

$\therefore$ The S.S. $= \{4.61 , -2.61\}$

[b] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+3)} \times \frac{x+3}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

, $n(x) = 1$

4

[a] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0 , 1\}$

, $n_1(x) = \frac{1}{x-1}$ } (1)

, $\because n_2(x) = \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 1\}$

, $n_2(x) = \frac{1}{x-1}$ } (2)

From (1) and (2) : $\therefore n_1 = n_2$

[b] $\because x = y + 1$ (1)

, $(x-y)^2 + y = 3$ (2)

Substituting from (1) in (2) :

$\therefore (y+1-y)^2 + y = 3$

$\therefore 1 + y = 3$ $\therefore y = 2$

Substituting in (1) : $\therefore x = 3$

$\therefore$ The S.S. $= \{(3 , 2)\}$

5

[a] $\because n(x) = \frac{x-2}{(x-2)(x-3)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x-3)}{x-2}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2 , 3\}$

, $n^{-1}(x) = x - 3$

[b] 1 The probability that the number on the card is divisible by $3 = \frac{5}{15} = \frac{1}{3}$

2 The probability that the number on the card is an odd number and divisible by $3 = \frac{3}{15} = \frac{1}{5}$

3 Beni Suef

1 1 a 2 d 3 c 4 a 5 b 6 d

2

[a] $\because n(x) = \frac{x(x+2)}{(x+2)(x-2)} + \frac{x+3}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2 , 2 , -3\}$

, $n(x) = \frac{x}{x-2} + \frac{1}{x-2} = \frac{x+1}{x-2}$

[b] $\because 2x + y = 1$ $\therefore y = 1 - 2x$ (1)

, $x + 2y = 5$ (2)

Substituting from (1) in (2) :

$\therefore x + 2(1 - 2x) = 5$ $\therefore x + 2 - 4x = 5$

$\therefore -3x + 2 = 5$ $\therefore -3x = 3$

$\therefore x = -1$

Substituting in (1) : $\therefore y = 3$

$\therefore$ The S.S. $= \{(-1 , 3)\}$

3

[a] $\because$ The domain of $f = \mathbb{R} - \{5\}$

$\therefore$ At $x = 5$ $\therefore x^2 - ax + 25 = 0$

$\therefore (5)^2 - 5a + 25 = 0$ $\therefore 25 - 5a + 25 = 0$

$\therefore 50 - 5a = 0$ $\therefore -5a = -50$

$\therefore a = 10$

[b] $\because 3x^2 = 5x - 1$ $\therefore 3x^2 - 5x + 1 = 0$

$\therefore a = 3$, $b = -5$, $c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{5 \pm \sqrt{13}}{6}$

$\therefore x \simeq 1.43$ or $x \simeq 0.23$

$\therefore$ The S.S. $= \{1.43 , 0.23\}$

4

[a] $\because n(x) = \frac{2(x-2)}{(x-2)(x^2+2x+4)} \div \frac{(x+2)}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -2\}$

, $n(x) = \frac{2}{x^2+2x+4} \times \frac{x^2+2x+4}{x+2} = \frac{2}{x+2}$

, $n(0) = \frac{2}{0+2} = 1$

[b] $\because x - y = 0$ $\therefore x = y$ (1)

, $x^2 + xy + y^2 = 27$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x^2 + x^2 = 27$ $\therefore 3x^2 = 27$

$\therefore x^2 = 9$ $\therefore x = 3$ or $x = -3$

Substituting in (1) : $\therefore y = 3$ or $y = -3$

$\therefore$ The S.S. $= \{(3 , 3) (-3 , -3)\}$

5

[a] $\because n_1(x) = \frac{x+5}{(x-1)(x+5)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{1, -5\}$

$, n_1(x) = \frac{1}{x-1}$

$, \because n_2(x) = \frac{2x-1}{(2x-1)(x-1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{1, \frac{1}{2}\}$

$, n_2(x) = \frac{1}{x-1}$

$\therefore n_1(x) = n_2(x)$ for all the values of $x \in \mathbb{R} - \{1, -5, \frac{1}{2}\}$

[b] 1 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.4 + 0.5 - 0.8 = 0.1$

2 The probability of non-occurrence of the event A

$= P(\acute{A}) = 1 - P(A) = 1 - 0.4 = 0.6$

4 El-Menia

1 1 d 2 a 3 a 4 d 5 d 6 c

2

[a] $\because 2y = 3 + x$ $\therefore x = 2y - 3$ (1)

$, 2x = 4 - y$ (2)

Substituting from (1) in (2) :

$\therefore 2(2y - 3) = 4 - y$ $\therefore 4y - 6 = 4 - y$

$\therefore 5y = 10$ $\therefore y = 2$

Substituting in (1) : $\therefore x = 1$

$\therefore$ The S.S. $= \{(1, 2)\}$

[b] $\because$ The domain of $n = \mathbb{R} - \{1, 3\}$

$\therefore$ At $x = 1$ $\therefore x^2 + bx + 3 = 0$

$\therefore (1)^2 + b + 3 = 0$ $\therefore b + 4 = 0$

$\therefore b = -4$

3

[a] $\because x + y = 0$ $\therefore y = -x$ (1)

$, x^2 + y^2 = 2$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + (-x)^2 = 2$ $\therefore x^2 + x^2 = 2$

$\therefore 2x^2 = 2$ $\therefore x^2 = 1$

$\therefore x = 1$ or $x = -1$

Substituting in (1) : $\therefore y = -1$ or $y = 1$

$\therefore$ The S.S. $= \{(1, -1), (-1, 1)\}$

[b] $\because n_1(x) = \frac{x-2}{(x-2)^2}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2\}$

$, n_1(x) = \frac{1}{x-2}$

$, \because n_2(x) = \frac{1}{x-2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{2\}$

$\therefore n_1(x) = n_2(x)$

for all the values of $x \in \mathbb{R} - \{2\}$

4

[a] $\because x^2 + 1 = 3x$ $\therefore x^2 - 3x + 1 = 0$

$\therefore a = 1$, $b = -3$, $c = 1$

$\therefore x = \frac{3 \pm \sqrt{(-3)^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{3 \pm \sqrt{5}}{2}$

$\therefore x \simeq 2.62$ or $x \simeq 0.38$

$\therefore$ The S.S. $= \{2.62, 0.38\}$

[b] $\because n(x) = \frac{x+2}{(x+2)(x-2)} + \frac{2x}{2(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -2\}$

$, n(x) = \frac{1}{x-2} + \frac{x}{x-2} = \frac{x+1}{x-2}$

5

[a] 1 $\because$ A , B are two mutually exclusive events

$\therefore P(A \cap B) = 0$

2 $\because P(A \cup B) = P(A) + P(B)$

$\therefore P(A) = P(A \cup B) - P(B) = 0.8 - 0.3 = 0.5$

[b] $\because n(x) = \frac{x(x+2)}{(x+2)(x^2 - 2x + 4)} \div \frac{2x}{x^2 - 2x + 4}$

$\therefore$ The domain of $n = \mathbb{R} - \{-2, 0\}$

$, n(x) = \frac{x}{x^2 - 2x + 4} \times \frac{x^2 - 2x + 4}{2x} = \frac{1}{2}$

5 Assiut

1 1 c 2 b 3 b 4 b 5 a 6 b

2

[a] $\because x(x+8) + 9 = 0$ $\therefore x^2 + 8x + 9 = 0$

$\therefore a = 1$, $b = 8$, $c = 9$

$\therefore x = \frac{-8 \pm \sqrt{(8)^2 - 4 \times 1 \times 9}}{2 \times 1} = \frac{-8 \pm 2\sqrt{7}}{2} = -4 \pm \sqrt{7}$

$\therefore x \simeq -1.4$ or $x \simeq -6.6$

$\therefore$ The S.S. $= \{-1.4, -6.6\}$

[b] $\because n(x) = \dfrac{(x+3)(x+1)}{(x-3)(x^2+3x+9)} \div \dfrac{x+3}{x^2+3x+9}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3\}$

, $n(x) = \dfrac{(x+3)(x+1)}{(x-3)(x^2+3x+9)} \times \dfrac{x^2+3x+9}{x+3} = \dfrac{x+1}{x-3}$

3

[a] $\because x - y = 1 \quad \therefore x = 1 + y \quad (1)$

, $x^2 + y^2 = 25 \quad (2)$

Substituting from (1) in (2) :

$\therefore (1+y)^2 + y^2 = 25 \quad \therefore 1 + 2y + y^2 + y^2 - 25 = 0$

$\therefore 2y^2 + 2y - 24 = 0 \quad \therefore y^2 + y - 12 = 0$

$\therefore (y-3)(y+4) = 0 \quad \therefore y = 3$ or $y = -4$

Substituting in (1) :

$\therefore x = 4$ or $x = -3$

$\therefore$ The S.S. $= \{(4, 3), (-3, -4)\}$

[b] $\because n(x) = \dfrac{x(x-1)}{(x-1)(x+1)} + \dfrac{x+5}{(x+1)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1, -1, -5\}$

, $n(x) = \dfrac{x}{x+1} + \dfrac{1}{x+1} = \dfrac{x+1}{x+1} = 1$

4

[a] $\because 2x - y = 3$ (multiplying by 2)

$\therefore 4x - 2y = 6 \quad (1)$

, $x + 2y = 4 \quad (2)$

Adding (1) and (2) : $\therefore 5x = 10 \quad \therefore x = 2$

Substituting in (1) : $\therefore y = 1$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] 1 $\because n(x) = \dfrac{x(x-2)}{(x-1)(x-2)}$

$\therefore n^{-1}(x) = \dfrac{(x-1)(x-2)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2, 1\}$

, $n^{-1}(x) = \dfrac{x-1}{x}$

2 $n^{-1}(2)$ is undefined because $2 \notin$ the domain of n^{-1}

5

[a] $\because n_1(x) = \dfrac{x-4}{(x-2)(x-3)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, 3\}$

, $\because n_2(x) = \dfrac{2x}{x(x-3)(x+3)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 3, -3\}$

$\therefore$ The common domain $= \mathbb{R} - \{0, 2, 3, -3\}$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{8} = \dfrac{17}{24}$

2 $\because A, B$ are mutually exclusive events

$\therefore P(A \cap B) = 0$

$\therefore P(A \cup B) = P(A) + P(B) = \dfrac{1}{2} + \dfrac{1}{3} = \dfrac{5}{6}$

6 Qena

1 1 a 2 c 3 d 4 b 5 b 6 c

2

[a] $\because x - y = 2 \quad \therefore x = 2 + y \quad (1)$

, $x^2 + y^2 = 20 \quad (2)$

Substituting from (1) in (2) :

$\therefore (2+y)^2 + y^2 = 20 \quad \therefore 4 + 4y + y^2 + y^2 - 20 = 0$

$\therefore 2y^2 + 4y - 16 = 0 \quad \therefore y^2 + 2y - 8 = 0$

$\therefore (y+4)(y-2) = 0$

$\therefore y = -4$ or $y = 2$

Substituting in (1) : $\therefore x = -2$ or $x = 4$

$\therefore$ The S.S. $= \{(-2, -4), (4, 2)\}$

[b] $\because n(x) = \dfrac{x+1}{(x-2)(x+1)} \times \dfrac{(x-2)(x+5)}{(3x+1)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -1, -\frac{1}{3}, -5\}$

, $n(x) = \dfrac{1}{3x+1}$

, $n(-1)$ is undefined because $-1 \notin$ the domain of n

3

[a] $\because x^2 - 4x + 1 = 0$

$\therefore a = 1 \ , \ b = -4 \ , \ c = 1$

$\therefore x = \dfrac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2 \times 1} = \dfrac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$

$\therefore x \simeq 3.73$ or $x \simeq 0.27$

$\therefore$ The S.S. $= \{3.73, 0.27\}$

[b] $\because n(x) = \dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \dfrac{(x-3)(x+3)}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$

, $n(x) = \dfrac{1}{x-2} + \dfrac{x-3}{x-2} = \dfrac{x-2}{x-2} = 1$

4

[a] Let the measures of the two angles be x, y

$\therefore x - y = 50° \quad (1)$

$\therefore x + y = 90°$ (2)

Adding (1) and (2) : $\therefore 2x = 140°$

$\therefore x = 70°$

Substituting in (1) : $\therefore y = 20°$

$\therefore$ The measures of the two angles are 70° and 20°

[b] $\because n_1(x) = \dfrac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0, 1\}$

$, n_1(x) = \dfrac{1}{x-1}$ } (1)

$, \because n_2(x) = \dfrac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$

$, n_2(x) = \dfrac{1}{x-1}$ } (2)

From (1) and (2) : $\therefore n_1 = n_2$

5

[a] 1 $\because$ The domain of $n = \mathbb{R} - \{1, -1\}$

$\therefore$ At $x = 1$ $\therefore x^2 - k = 0$

$\therefore (1)^2 - k = 0$ $\therefore k = 1$

2 $\because n(x) = \dfrac{(x-2)(x+1)}{x^2-1} = \dfrac{(x-2)(x+1)}{(x+1)(x-1)}$

$\therefore n^{-1}(x) = \dfrac{(x+1)(x-1)}{(x-2)(x+1)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2, 1, -1\}$

$, n^{-1}(x) = \dfrac{x-1}{x-2}$

[b] 1 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.3 + 0.6 - 0.7 = 0.2$

2 $P(A - B) = P(A) - P(A \cap B) = 0.3 - 0.2 = 0.1$

3 $P(\hat{A}) = 1 - P(A) = 1 - 0.3 = 0.7$

7 Luxor

1 1 c 2 b 3 d 4 a 5 c 6 b

2

[a] $\because 2x - y = 3$ $\therefore y = 2x - 3$ (1)

$, x + 2y = 4$ (2)

Substituting from (1) in (2) :

$\therefore x + 2(2x - 3) = 4$ $\therefore x + 4x - 6 = 4$

$\therefore 5x = 10$ $\therefore x = 2$

Substituting in (1) : $\therefore y = 1$

$\therefore$ The S.S. $= \{(2, 1)\}$

[b] $\because n(x) = \dfrac{x(x+2)}{(x-3)(x^2+3x+9)} \div \dfrac{x+2}{x^2+3x+9}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -2\}$

$, n(x) = \dfrac{x(x+2)}{(x-3)(x^2+3x+9)} \times \dfrac{x^2+3x+9}{x+2} = \dfrac{x}{x-3}$

$, n(2) = \dfrac{2}{2-3} = -2$

$, n(-2)$ is undefined because $-2 \notin$ the domain of n

3

[a] $\because x^2 - 2x = 6$ $\therefore x^2 - 2x - 6 = 0$

$\therefore a = 1$, $b = -2$, $c = -6$

$\therefore x = \dfrac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times -6}}{2 \times 1} = \dfrac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$= 1 \pm 2.65$

$\therefore x = 3.65$ or $x = -1.65$

$\therefore$ The S.S. $= \{3.65, -1.65\}$

[b] 1 $n(x) = \dfrac{(x-2)(x+2)}{(x-2)(x-3)} = \dfrac{x+2}{x-3}$

2 The domain of $n = \mathbb{R} - \{2, 3\}$

3 $z(n) = \{-2\}$

4

[a] $\because y - x = 2$

$\therefore y = 2 + x$ (1)

$, x^2 + xy - 4 = 0$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x(2 + x) - 4 = 0$

$\therefore x^2 + 2x + x^2 - 4 = 0$

$\therefore 2x^2 + 2x - 4 = 0$

$\therefore x^2 + x - 2 = 0$

$\therefore (x-1)(x+2) = 0$

$\therefore x = 1$ or $x = -2$

Substituting in (1) : $\therefore y = 3$ or $y = 0$

$\therefore$ The S.S. $= \{(1, 3), (-2, 0)\}$

[b] $\because n(x) = \dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \dfrac{(x-3)(x+3)}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$

$, n(x) = \dfrac{1}{x-2} + \dfrac{x-3}{x-2} = \dfrac{x-2}{x-2} = 1$

5

[a] Let the measures of the two angles be x , y

$\therefore x - y = 50°$ (1)

, $x + y = 90°$ (2)

Adding (1) and (2) : $\therefore 2x = 140°$

$\therefore x = 70°$

Substituting in (1) : $\therefore y = 20°$

$\therefore$ The measures of the two angles are 70° and 20°

[b] 1 $\because$ A , B are mutually exclusive events

$\therefore P(A \cap B) = 0 \quad \therefore P(A \cup B) = P(A) + P(B)$

$\therefore 0.8 = 0.5 + x \quad \therefore x = 0.8 - 0.5 = 0.3$

2 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore 0.8 = 0.5 + x - 0.1$

$\therefore 0.8 = 0.4 + x \quad \therefore x = 0.8 - 0.4 = 0.4$

8 New Valley

1 1 b 2 c 3 b

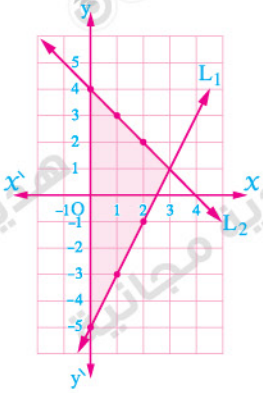
4 d 5 d 6 c

2

[a] $y = 2x - 5$, $y = 4 - x$

x	0	1	2
y	-5	-3	-1

x	0	1	2
y	4	3	2



From the graph : $\therefore$ The S.S. $= \{(3 , 1)\}$

The area of the triangle $= \frac{1}{2} \times 9 \times 3 = 13.5$ square units

[b] $\because n(x) = \frac{x+3}{x^2+2x+4} \times \frac{(x-2)(x^2+2x+4)}{(x-2)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

, $n(x) = 1$

3

[a] 1 $\because n(x) = \frac{x(x-3)}{(x-3)(x-1)}$

$\therefore n^{-1}(x) = \frac{(x-3)(x-1)}{x(x-3)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0 , 3 , 1\}$

, $n^{-1}(x) = \frac{x-1}{x}$

2 $\because n^{-1}(x) = 2$

$\therefore \frac{x-1}{x} = 2 \quad \therefore 2x = x - 1 \quad \therefore x = -1$

[b] $\because y - x = 2 \quad \therefore x = y - 2$ (1)

, $xy + y^2 = 4$ (2)

Substituting from (1) in (2) : $\therefore y(y-2) + y^2 = 4$

$\therefore y^2 - 2y + y^2 - 4 = 0 \quad \therefore 2y^2 - 2y - 4 = 0$

$\therefore y^2 - y - 2 = 0 \quad \therefore (y-2)(y+1) = 0$

$\therefore y = 2$ or $y = -1$

Substituting in (1) : $\therefore x = 0$ or $x = -3$

$\therefore$ The S.S. $= \{(0 , 2) , (-3 , -1)\}$

4

[a] $\because 2x^2 = 1 + 5x \quad \therefore 2x^2 - 5x - 1 = 0$

$\therefore a = 2 \ , \ b = -5 \ , \ c = -1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times -1}}{2 \times 2} = \frac{5 \pm \sqrt{33}}{4}$

$\therefore x \simeq 2.7$ or $x \simeq -0.2$

$\therefore$ The S.S. $= \{2.7 , -0.2\}$

[b] $\because n(x) = \frac{x(x-1)}{(x-1)(x+1)} + \frac{x+5}{(x+5)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , -5\}$

, $n(x) = \frac{x}{x+1} + \frac{1}{x+1} = \frac{x+1}{x+1} = 1$

5

[a] $\because n_1(x) = \frac{x^2}{x^2(x-1)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{0 , 1\}$

, $n_1(x) = \frac{1}{x-1}$ } (1)

, $\because n_2(x) = \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{0 , 1\}$

, $n_2(x) = \frac{1}{x-1}$ } (2)

From (1) and (2) : $\therefore n_1 = n_2$

[b] 1 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{1}{2} + \frac{1}{3} - \frac{17}{24} = \frac{1}{8}$

2 $P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

9 South Sinai

1 1 d 2 a 3 b 4 b 5 a 6 c

2

[a] $\because n(x) = \frac{x-3}{(x-3)(x-4)} - \frac{4}{x(x-4)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, 3, 4\}$

, $n(x) = \frac{1}{x-4} - \frac{4}{x(x-4)} = \frac{x-4}{x(x-4)} = \frac{1}{x}$

[b] $\because 2x^2 - 5x + 1 = 0$

$\therefore a = 2$, $b = -5$, $c = 1$

$\therefore x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{5 \pm \sqrt{17}}{4}$

$\therefore x \simeq 2.28$ or $x \simeq 0.22$

$\therefore$ The S.S. $= \{2.28, 0.22\}$

3

[a] $\because n_1(x) = \frac{2x}{2(x+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{-2\}$ } (1)

, $n_1(x) = \frac{x}{x+2}$

, $\because n_2(x) = \frac{x(x+2)}{(x+2)^2}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{-2\}$ } (2)

, $n_2(x) = \frac{x}{x+2}$

From (1) and (2) : $\therefore n_1 = n_2$

[b] $\because x - y = 0$ $\therefore x = y$ (1)

, $x^2 + xy + y^2 = 27$ (2)

Substituting from (1) in (2) :

$\therefore x^2 + x^2 + x^2 = 27$ $\therefore 3x^2 = 27$

$\therefore x^2 = 9$

$\therefore x = 3$ or $x = -3$

Substituting in (1) : $\therefore y = 3$ or $y = -3$

$\therefore$ The S.S. $= \{(3, 3), (-3, -3)\}$

4

[a] $\because 2x - y = 3$ $\therefore y = 2x - 3$ (1)

, $x + 2y = 4$ (2)

Substituting from (1) in (2) :

$\therefore x + 2(2x - 3) = 4$ $\therefore x + 4x - 6 = 4$

$\therefore 5x = 10$ $\therefore x = 2$

Substituting in (1) : $\therefore y = 1$

[b] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x+3)} \times \frac{x+3}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -3\}$

, $n(x) = 1$

5

[a] 1 $\because n(x) = \frac{x(x-2)}{(x-2)(x-1)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x-1)}{x(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, 2, 1\}$

, $n^{-1}(x) = \frac{x-1}{x}$

2 $\because n^{-1}(x) = 3$ $\therefore \frac{x-1}{x} = 3$

$\therefore 3x = x - 1$ $\therefore 2x = -1$

$\therefore x = -\frac{1}{2}$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.6 - 0.2 = 0.7$

2 $P(\bar{B}) = 1 - P(B) = 1 - 0.6 = 0.4$

10 Red Sea

1 1 c 2 d 3 b 4 c 5 a 6 b

2

[a] $\because x + y = 4$ (1)

, $2x - y = 2$ (2)

Adding (1) and (2) : $\therefore 3x = 6$ $\therefore x = 2$

Substituting in (1) : $\therefore y = 2$

$\therefore$ The S.S. $= \{(2, 2)\}$

[b] $\because n_1(x) = \frac{(x+2)(x+1)}{(x-2)(x+2)}$

$\therefore$ The domain of $n_1 = \mathbb{R} - \{2, -2\}$

, $n_1(x) = \frac{x+1}{x-2}$

, $\because n_2(x) = \frac{(x-1)(x+1)}{(x-1)(x-2)}$

$\therefore$ The domain of $n_2 = \mathbb{R} - \{1, 2\}$

, $n_2(x) = \frac{x+1}{x-2}$

$\therefore n_1(x) = n_2(x)$ for all the values

of $x \in \mathbb{R} - \{1, 2, -2\}$

3

[a] $\because n(x) = \frac{(x-2)(x^2+2x+4)}{(x-2)(x-1)} \times \frac{x+1}{x^2+2x+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, 1\}$

, $n(x) = \frac{x+1}{x-1}$

[b] $\because x^2 - 2x - 6 = 0$

$\therefore a = 1 \quad , \quad b = -2 \quad , \quad c = -6$

$\therefore x = \frac{2 \pm \sqrt{(-2)^2 - 4 \times 1 \times -6}}{2 \times 1} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$\therefore x \simeq 3.6 \quad$ or $\quad x \simeq -1.6$

$\therefore$ The S.S. $= \{3.6 , -1.6\}$

4

[a] $\because x - y = 1 \qquad \therefore x = 1 + y \qquad (1)$

$, x^2 - y^2 = 25 \qquad (2)$

Substituting from (1) in (2) :

$\therefore (1 + y)^2 - y^2 = 25$

$\therefore 1 + 2y + y^2 - y^2 = 25$

$\therefore 2y = 24 \qquad \therefore y = 12$

Substituting in (1) : $\therefore x = 13$

[b] $\because n(x) = \frac{x(x+1)}{(x-1)(x+1)} - \frac{x+5}{(x-1)(x+5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , -5\}$

$, n(x) = \frac{x}{x-1} - \frac{1}{x-1} = \frac{x-1}{x-1} = 1$

5

[a] $\because n(x) = \frac{2(x-2)}{(x-2)(x+2)}$

$\therefore n^{-1}(x) = \frac{(x-2)(x+2)}{2(x-2)}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2 , -2\}$

$, n^{-1}(x) = \frac{x+2}{2}$

[b] 1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.6 + 0.5 - 0.3 = 0.8$

2 $P(A - B) = P(A) - P(A \cap B) = 0.6 - 0.3 = 0.3$

3 $P(\grave{A}) = 1 - P(A) = 1 - 0.6 = 0.4$

1 Suez

1 1 b 2 c 3 a
4 b 5 b 6 d

2

[a] In the greater circle :

$\because \overline{ME} \perp \overline{AB}$ $\therefore$ E is the midpoint of $\overline{AB}$

$\therefore AE = BE$ (1)

In the smaller circle :

$\because \overline{ME} \perp \overline{CD}$ $\therefore$ E is the midpoint of $\overline{CD}$

$\therefore CE = DE$ (2)

Subtracting (2) from (1) : $\therefore AC = BD$ (Q.E.D.)

[b] $\because \overline{AB}$ is a diameter

$\therefore m(\angle C) = 90°$ (First req.)

, $\because m(\angle B) = \frac{1}{2} m(\overset{\frown}{AC}) = \frac{1}{2} \times 120° = 60°$

$\therefore$ In ΔACB : $m(\angle A) = 180° - (90° + 60°) = 30°$

(Second req.)

3

[a] $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

, $MX = MY$ $\therefore AB = CD$

, $\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore AB = CD = 2 \times 7 = 14$ cm. (The req.)

[b] In ΔABC :

$m(\angle C) = 180° - (70° + 60°) = 50°$

, $\because \overrightarrow{AD}$ is a tangent to the circle M at A

$\therefore m(\angle BAD)$ (tangency) $= m(\angle C)$ (inscribed) $= 50°$

(The req.)

4

[a] State by yourself.

[b] $\because \overrightarrow{CF}$ bisects $\angle DCE$

$\therefore m(\angle DCE) = 2 \times 50° = 100°$

, $\because \overline{AD} // \overrightarrow{BC}$, $\overleftrightarrow{CD}$ is a transversal

$\therefore m(\angle D) = m(\angle DCE) = 100°$ (alternate angles)

$\therefore m(\angle B) + m(\angle D) = 80° + 100° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 45° = 90°$

(central and inscribed angles subtended by $\overset{\frown}{AB}$)

(First req.)

, $\because MA = MB = r$

$\therefore$ In ΔAMB :

$m(\angle MAB) = m(\angle MBA) = \frac{180° - 90°}{2} = 45°$

(Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore$ In ΔABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180 - 40°}{2} = 70°$

, $\because$ CEDB is a cyclic quadrilateral

$\therefore m(\angle DBC) = 180° - 110° = 70°$

$\therefore m(\angle ABC) = m(\angle DBC)$ (Q.E.D.)

2 Kafr El-Sheikh

1 1 b 2 c 3 d
4 b 5 c 6 a

2

[a] $\because XY = XZ$

, $\overrightarrow{MA} \perp \overline{XY}$, $\overrightarrow{MB} \perp \overline{XZ}$

$\therefore MA = MB$

, $\because MC = MD = r$

Subtracting : $\therefore AC = BD$ (Q.E.D.)

[b] $\because$ A is the midpoint of $\overline{XB}$

$\therefore \overline{MA} \perp \overline{XB}$ $\therefore m(\angle MAB) = 90°$

, $\because \overline{YZ}$ is a tangent-segment to the circle at Y

$\therefore \overline{MY} \perp \overline{YZ}$

$\therefore m(\angle MYZ) = 90°$

$\therefore m(\angle MAB) + m(\angle MYZ) = 90° + 90° = 180°$

$\therefore$ MAZY is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle.

$\therefore AD = AF = 3$ cm.

, $\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle.

$\therefore$ BD = BE = 4 cm.

, $\because$ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle.

$\therefore$ CE = CF = 5 cm.

$\therefore$ The perimeter of Δ ABC = 3 + 3 + 4 + 4 + 5 + 5
= 24 cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ C) = 180° – 100° = 80°

$\therefore$ In Δ CBD :

$\because$ BC = DC

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CDB) = $\frac{180° - 80°}{2}$
= 50° (The req.)

4

[a] $\because$ m ($\overset{\frown}{ABC}$) = m ($\angle$ CMA) = 100°

$\therefore$ m ($\overset{\frown}{AC}$ the major) = 360° – 100° = 260°

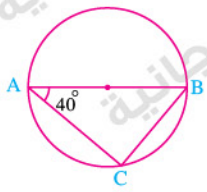
$\therefore$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$ the major)
= $\frac{1}{2}$ × 260° = 130° (The req.)

[b] $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ In Δ ACB :

m ($\angle$ B) = 180° – (90° + 40°)
= 50°



(The req.)

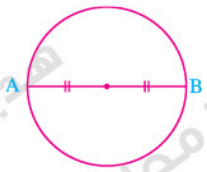
5

[a] $\because$ $\overrightarrow{BC}$ is a tangent to the circle at B

$\therefore$ m ($\angle$ D) (inscribed) = m ($\angle$ ABC) (tangency) = 70°
(First req.)

$\therefore$ m ($\overset{\frown}{AB}$ the minor) = 2 m ($\angle$ D) = 2 × 70°
= 140° (Second req.)

[b]



We can draw one circle.

3 El-Fayoum

1 1 b 2 c 3 b
4 c 5 d 6 c

2

[a] $\because$ D is the midpoint of $\overline{OH}$

$\therefore$ $\overline{MD} \perp \overline{OH}$ $\therefore$ m ($\angle$ MDA) = 90°

, $\because$ $\overrightarrow{AB}$ is a tangent to the big circle at B

$\therefore$ $\overline{MB} \perp \overrightarrow{AB}$ $\therefore$ m ($\angle$ MBA) = 90°

$\therefore$ From the quadrilateral ABMD :

m ($\angle$ DMB) = 360° – (90° + 90° + 40°)
= 140° (The req.)

[b] In Δ ZLY : $\because$ LZ = ZY

$\therefore$ m ($\angle$ ZYL) = m ($\angle$ ZLY) = 40°

$\therefore$ m ($\angle$ Z) = 180° – 2 × 40° = 100°

$\therefore$ m ($\angle$ X) + m ($\angle$ Z) = 80° + 100° = 180°

$\therefore$ XYZL is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because$ AB = AC

, $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore$ MD = MH , $\because$ MX = MY = r

Subtracting : $\therefore$ DX = HY (Q.E.D.)

[b] In Δ ABC :

$\because$ m ($\angle$ CAB) + m ($\angle$ C) + m ($\angle$ B) = 180°

$\therefore$ 60° + 5 $\mathcal{X}$° + 3 $\mathcal{X}$° = 180°

$\therefore$ 8 $\mathcal{X}$ = 180° – 60° = 120°

$\therefore$ $\mathcal{X}$ = 15° (First req.)

$\therefore$ m ($\angle$ C) = 5 × 15° = 75°

$\therefore$ m ($\angle$ OAB) = m ($\angle$ C)

$\therefore$ $\overrightarrow{AO}$ is a tangent to the circle which passes through the points A , B and C (Second req.)

4

[a] $\because$ m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{CD}$) = 70°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD) = $\frac{1}{2}$ × 70° = 35°

$\therefore$ In Δ ACD :

m ($\angle$ ADC) = 180° – (35° + 35°) = 110° (First req.)

, $\because$ $\overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ m ($\angle$ DCB) = 90° + 35° = 125° (Second req.)

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{BC}$) – m ($\overset{\frown}{HD}$)]

$\therefore$ 30° = $\frac{1}{2}$ [80° – m ($\overset{\frown}{HD}$)]

$\therefore$ 60° = 80° – m ($\overset{\frown}{HD}$)

$\therefore$ m ($\overset{\frown}{HD}$) = 80° – 60° = 20° (The req.)

5

[a] $\because$ $\overrightarrow{CH}$ bisects $\angle$ DCO

$\therefore$ m ($\angle$ DCO) = 2 × 55° = 110°

, $\because$ $\overrightarrow{AD}$ // $\overrightarrow{BC}$, $\overleftrightarrow{DC}$ is a transversal

$\therefore m(\angle D) = m(\angle DCO) = 110°$ (alternate angles)

$\therefore m(\angle B) + m(\angle D) = 70° + 110° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore$ In ΔABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180° - 40°}{2} = 70°$

, $\because$ BCXY is a cyclic quadrilateral

$\therefore m(\angle CBY) = 180° - 110° = 70°$

$\therefore m(\angle ABC) = m(\angle CBY)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABY$ (Q.E.D.)

4 Beni Suef

1 1 c 2 b 3 d 4 b 5 a 6 a

2

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180° - 80° = 100°$

$\therefore$ In ΔABD :

$m(\angle ABD) = 180° - (30° + 100°) = 50°$ (The req.)

[b] $\because m(\angle DBC) = \frac{1}{2} m(\angle DMC) = \frac{1}{2} \times 140° = 70°$

(inscribed and central angles subtended by $\overset{\frown}{CD}$)

$\therefore m(\angle DBA) = 180° - 70° = 110°$

$\therefore$ In ΔABD : $\because AB = BD$

$\therefore m(\angle BAD) = m(\angle BDA) = \frac{180° - 110°}{2} = 35°$

(The req.)

3

[a] $\because \overline{XY} // \overline{AB}$, $\overleftrightarrow{CB}$ is a transversal

$\therefore m(\angle CYX) = m(\angle B)$ (corresponding angles)

, $\because m(\angle B)$ (inscribed) $= m(\angle ACD)$ (tangency)

$\therefore m(\angle CYX) = m(\angle XCD)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle passing through the vertices of ΔXYC (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle at D , E

$\therefore \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, $\because$ MD = ME (radii lengths of the smaller circle)

$\therefore AB = AC$ (First req.)

$\therefore$ In ΔABC :

$m(\angle C) = m(\angle B) = 65°$

$\therefore m(\angle BAC) = 180° - 2 \times 65° = 50°$ (Second req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB) = \frac{180° - 40°}{2} = 70°$

, $\because \overline{AB}$ is a tangent-segment at B

$\therefore \overline{MB} \perp \overline{AB}$ $\therefore m(\angle MBA) = 90°$

$\therefore m(\angle CBD) = 90° - 70° = 20°$ (The req.)

[b] $\because$ D is the midpoint of $\overline{CE}$

$\therefore \overline{MD} \perp \overline{CE}$ $\therefore m(\angle MDC) = 90°$ (1)

, $\because \overleftrightarrow{BC}$ is a tangent to the circle at C

$\therefore \overline{MC} \perp \overleftrightarrow{BC}$

$\therefore m(\angle MCB) = 90°$

, $\because \overline{AB} // \overline{MC}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle CBA) + m(\angle MCB) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CBA) = 180° - 90° = 90°$ (2)

From (1) and (2) :

$\therefore m(\angle MDC) + m(\angle CBA) = 90° + 90° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because AB = CD$

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ (1)

, $\because \overline{AB} // \overline{CD}$

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{AC})$ (2)

Adding (1) and (2) :

$\therefore m(\overset{\frown}{ABD}) = m(\overset{\frown}{ACD})$

$\therefore \overline{AD}$ is a diameter in the circle. (Q.E.D.)

[b] $\because \overleftrightarrow{AB}$ is a tangent to the circle at A

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

$\therefore m(\angle MAB) = 90°$

, $\because \angle AMC$ is exterior of ΔAMB

$\therefore m(\angle AMC) = m(\angle MAB) + m(\angle B)$

$= 90° + 30° = 120°$ (First req.)

$\therefore$ In ΔAMC :

$\because MC = MA = r$

$\therefore m(\angle MCA) = m(\angle MAC) = \frac{180° - 120°}{2} = 30°$

$\therefore$ In ΔABC : $m(\angle B) = m(\angle ACB)$

$\therefore AB = AC = 6$ cm. (Second req.)

5 Souhag

1 1 c 2 a 3 b 4 c 5 a 6 c

2

[a] ∵ ABCD is a cyclic quadrilateral

∴ m (∠ A) = 180° – 70° = 110°

∴ In Δ ABD :

m (∠ ABD) = 180° – (30° + 110°) = 40° (The req.)

[b] ∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$

, ∵ Y is the midpoint of $\overline{CD}$ ∴ $\overline{MY} \perp \overline{CD}$

, ∵ XE = YF , ME = MF = r

Subtracting : ∴ MX = MY

∴ AB = CD (Q.E.D.)

3

[a] ∵ $\overline{XY} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

∴ m (∠ AYX) = m (∠ C) (corresponding angles)

, ∵ m (∠ C) (inscribed) = m (∠ BAD) (tangency)

∴ m (∠ AYX) = m (∠ XAD)

∴ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] ∵ $m(\angle A) = \frac{1}{2}[m(\overset{\frown}{EC}) - m(\overset{\frown}{DB})]$

∴ $20° = \frac{1}{2}[m(\overset{\frown}{EC}) - 40°]$

∴ $40° = m(\overset{\frown}{EC}) - 40°$

∴ $m(\overset{\frown}{EC}) = 40° + 40° = 80°$

, ∵ $\overline{BC}$ is a diameter

∴ $m(\overset{\frown}{BC}) = 180°$

∴ $m(\overset{\frown}{ED}) = 180° - (80° + 40°) = 60°$ (The req.)

4

[a] ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

∴ AB = AC

∴ Δ ABC is an isosceles triangle

, ∵ m (∠ CBA) (tangency) = m (∠ D) (inscribed) = 60°

∴ Δ ABC is an equilateral triangle.

∴ The perimeter of Δ ABC = 3 × 10 = 30 cm. (The req.)

[b] ∵ E is the midpoint of $\overline{AC}$

∴ $\overline{ME} \perp \overline{AC}$ ∴ m (∠ MED) = 90°

, ∵ $\overline{BD}$ is a tangent-segment to the circle M at B

∴ $\overline{MB} \perp \overline{BD}$ ∴ m (∠ MBD) = 90°

∴ m (∠ MED) + m (∠ MBD) = 90° + 90° = 180°

∴ MEDB is a cyclic quadrilateral. (Q.E.D.)

5

[a] In Δ ABM :

∵ MA = MB = r

∴ m (∠ MAB) = m (∠ MBA) = 40°

∴ m (∠ AMB) = 180° – 2 × 40° = 100°

∴ $m(\angle ACB) = \frac{1}{2} m(\angle AMB) = \frac{1}{2} \times 100° = 50°$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

, ∵ $m(\overset{\frown}{AC}) = m(\overset{\frown}{BC})$ ∴ AC = BC

∴ In Δ ABC :

$m(\angle BAC) = m(\angle ABC) = \frac{180° - 50°}{2} = 65°$

∴ m (∠ CAM) = 65° – 40° = 25° (The req.)

[b] 1 ∵ $r_1 - r_2 = 8 - 6 = 2$ cm.

∴ $MN = r_1 - r_2$

∴ The two circles are touching internally.

2 ∵ $r_1 + r_2 = 8 + 6 = 14$ cm. , $r_1 - r_2 = 2$ cm.

∴ $r_1 - r_2 < MN < r_1 + r_2$

∴ The two circles are intersecting.

3 $MN \notin]0 , \infty[$

∴ The two circles are concentric.

6 Aswan

1 1 c 2 d 3 a 4 b 5 c 6 a

2

[a] ∵ $\overline{AB}$ is a diameter

∴ $m(\overset{\frown}{AC}) = 180° - 128° = 52°$

∴ $m(\angle B) = \frac{1}{2} m(\overset{\frown}{AC}) = \frac{1}{2} \times 52° = 26°$ (The req.)

[b] ∵ $\overline{XY} // \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

∴ m (∠ DBX) = m (∠ YXB) (alternate angles) (1)

, ∵ m (∠ C) (inscribed) = m (∠ ABD) (tangency) (2)

From (1) and (2) :

∴ m (∠ C) = m (∠ YXB)

∴ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] ∵ Δ MAB is an equilateral triangle.

∴ m (∠ AMB) = 60°

, ∵ $\overrightarrow{BC}$ is a tangent to the circle at B

∴ $m(\angle ABC)$ (tangency) $= \frac{1}{2} m(\angle AMB)$ (central)

$= \frac{1}{2} \times 60° = 30°$ (The req.)

[b] In Δ ABC : ∵ AB = AC

∴ m (∠ B) = m (∠ C)

∴ $m(\overset{\frown}{EB}) = m(\overset{\frown}{CD})$

By subtracting $m(\overset{\frown}{ED})$ from both sides

∴ $m(\overset{\frown}{DB}) = m(\overset{\frown}{CE})$

∴ BD = CE (Q.E.D.)

4

[a] ∵ $m(\angle E) = \frac{1}{2}[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})]$

∴ $30° = \frac{1}{2}[80° - m(\overset{\frown}{BD})]$

∴ $60° = 80° - m(\overset{\frown}{BD})$

∴ $m(\overset{\frown}{BD}) = 80° - 60° = 20°$

, ∵ $\overline{AB}$ is a diameter ∴ $m(\overset{\frown}{AB}) = 180°$

∴ $m(\overset{\frown}{CD}) = 180° - (80° + 20°) = 80°$ (The req.)

[b] In Δ ABD : ∵ AB = AD

∴ $m(\angle ABD) = m(\angle ADB) = 30°$

∴ $m(\angle A) = 180° - 2 \times 30° = 120°$

∴ $m(\angle A) = m(\angle DCE)$

∴ ABCD is a cyclic quadrilateral (Q.E.D.)

5

[a] In Δ ABM : ∵ MA = MB = r

∴ $m(\angle MAB) = m(\angle MBA) = 50°$

∴ $m(\angle M) = 180° - 2 \times 50° = 80°$

∴ $m(\angle C) = \frac{1}{2} m(\angle M) = \frac{1}{2} \times 80° = 40°$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

(The req.)

[b] ∵ E is the midpoint of $\overline{BC}$

∴ $\overline{ME} \perp \overline{BC}$ ∴ $m(\angle MEA) = 90°$

, ∵ $\overrightarrow{AD}$ is a tangent to the circle at D

∴ $\overline{MD} \perp \overrightarrow{AD}$

∴ $m(\angle MDA) = 90°$

∴ From the quadrilateral ADME :

$m(\angle DME) = 360° - (90° + 90° + 56°)$

$= 124°$ (The req.)

7 New Valley

1 1 d 2 a 3 c 4 d 5 b 6 c

2

[a] ∵ $m(\overset{\frown}{BC}) = 2\, m(\angle CAB) = 2 \times 30° = 60°$ (1)

, $\overline{AB}$ is a diameter

, D is the midpoint of $\overset{\frown}{AC}$

∴ $m(\overset{\frown}{AD}) = m(\overset{\frown}{CD}) = \frac{180° - 60°}{2} = 60°$ (2)

(First req.)

From (1) and (2) : ∴ $m(\overset{\frown}{AD}) = m(\overset{\frown}{BC})$

∴ $m(\angle ACD) = m(\angle CAB)$ and they are alternate angles

∴ $\overline{AB} // \overline{CD}$ (Second req.)

[b] ∵ BCDE is a cyclic quadrilateral

∴ $m(\angle CBE) = 180° - 125° = 55°$

, ∵ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

∴ AB = AC

∴ In Δ ABC : $m(\angle ACB) = m(\angle ABC)$

$= \frac{180° - 70°}{2} = 55°$

∴ $m(\angle BEC)$ (inscribed) $= m(\angle ACB)$ (tangency)

$= 55°$

∴ $m(\angle CBE) = m(\angle BEC)$

∴ In Δ CBE : CB = CE (Q.E.D. 1)

, ∵ $m(\angle CBE) = m(\angle ACB) = 55°$

and they are alternate angles

∴ $\overrightarrow{AC} // \overline{BE}$ (Q.E.D. 2)

3

[a] State by yourself.



[b] ∵ $\overline{CD}$ is a diameter

∴ $m(\angle DXC) = 90°$

, ∵ $\overline{CD} \perp \overline{AB}$

∴ $m(\angle YEC) = 90°$

∴ $m(\angle YXC) + m(\angle YEC) = 90° + 90° = 180°$

∴ XYEC is a cyclic quadrilateral (Q.E.D. 1)

∴ $m(\angle DYE) = m(\angle C)$

, ∵ $m(\angle DBX) = m(\angle C)$

(two inscribed angles subtended by $\overset{\frown}{XD}$)

∴ $m(\angle DYB) = m(\angle DBX)$ (Q.E.D. 2)

4

[a] ∵ $\overline{CF}$, $\overline{CE}$ are two tangent-segments to the circle

∴ CF = CE = 3 cm.

∵ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle.

∴ AD = AF

∵ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle.

∴ BD = BE

, ∵ AD + BD = 9 cm. ∴ AF + BE = 9 cm.

∴ The perimeter of Δ ABC

= 9 + 9 + 3 + 3 = 24 cm. (The req.)

[b] ∵ $m(\angle ACB) = \frac{1}{2} m(\angle AMB) = \frac{1}{2} \times 120° = 60°$ (1)

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

, ∵ $\overrightarrow{CD} // \overline{AB}$ ∴ $m(\overset{\frown}{CA}) = m(\overset{\frown}{CB})$

∴ CA = CB (2)

From (1) and (2) :

∴ Δ ABC is an equilateral triangle. (Q.E.D.)

5

[a] $\because MF = ME = r$, $XF = YE$

Subtracting : $\therefore MX = MY$

, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore AB = CD$ (Q.E.D. 1)

$\because \overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$

, $\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$

, $\because AB = CD$ $\therefore AX = CY$

$\therefore$ In $\Delta\Delta AXF$, CYE : $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$

$\therefore AF = CE$ (Q.E.D. 2)

[b] In ΔAMB : $\because AM = BM = r$

$\therefore m(\angle MBA) = m(\angle MAB) = 20°$

, $\because$ C is the midpoint of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$ $\therefore m(\angle MCB) = 90°$

$\therefore$ In ΔBCM :

$m(\angle BMC) = 180° - (90° + 20°) = 70°$

$\therefore m(\angle BED) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 70° = 35°$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

(First req.)

$\because m(\angle MAB) = m(\angle MBA) = 20°$

$\therefore m(\angle AMB) = 180° - (20° + 20°) = 140°$

$\therefore m(\overset{\frown}{ADB}) = m(\angle AMB) = 140°$ (Second req.)

8 South Sinai

1 1 b 2 b 3 c 4 d 5 a 6 d

2

[a] $m(\angle BAD) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150° = 75°$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

(First req.)

$\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BCD) = 180° - 75° = 105°$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$ $\therefore 2X - 1 = X + 2$

$\therefore X = 3$ cm. (First req.)

$\therefore AB = AC = 5$ cm. and $BC = 4$ cm.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5 + 4 = 14$ cm.

(Second req.)

3

[a] State by yourself.

[b] In ΔABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle A) = 180° - 2 \times 30° = 120°$

$\therefore m(\angle A) + m(\angle C) = 120° + 60° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90°$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

$\therefore m(\angle MYA) = 90°$

From the quadrilateral AXMY :

$m(\angle DME) = 360° - (90° + 90° + 70°) = 110°$

(First req.)

, $\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$

, $\because MD = ME = r$

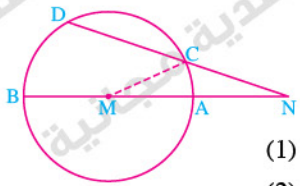
By subtracting : $\therefore XD = YE$ (Second req.)

[b] $\because m(\angle CAE) = 180° - 130° = 50°$

$\therefore m(\angle ABC)$ (inscribed) $= m(\angle CAE)$ (tangency)

$= 50°$ (The req.)

5



[a] **Const. :** Draw $\overline{MC}$

Proof : In ΔNMC :

$\because CN + MC > NM$ (1)

, $MC = MA = r$ (2)

Subtracting (2) from (1) :

$\therefore CN > AN$ (Q.E.D.)

[b] $\because \overline{DE} // \overline{BC}$ $\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{EC})$

$\therefore m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

9 North Sinai

1 1 b 2 b 3 c
4 a 5 a 6 c

2

[a] ∵ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M
∴ AB = AC ∴ 2 $\mathcal{X}$ – 3 = 15
∴ 2 $\mathcal{X}$ = 18 ∴ $\mathcal{X}$ = 9 cm.
, ∵ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the circle N
∴ AC = AD
∴ y – 2 = 15 ∴ y = 17 cm. (The req.)

[b] ∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$
, ∵ Y is the midpoint of $\overline{AC}$ ∴ $\overline{MY} \perp \overline{AC}$
, ∵ AB = AC ∴ MX = MY
, ∵ MD = ME = r ∴ XD = YE
(Q.E.D.)

3

[a] ∵ m $(\overset{\frown}{BC})$ = m (∠ BMC) = 60°
∴ m $(\overset{\frown}{BD})$ = m $(\overset{\frown}{BC})$ = 60°
∴ m (∠ BAD) = $\frac{1}{2}$ m $(\overset{\frown}{BD})$ = $\frac{1}{2}$ × 60° = 30°
(The req.)

[b] ∵ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C
∴ m (∠ ABC) = m (∠ ACB)
, ∵ m (∠ ABC) (tangency)
= m (∠ BDC) (inscribed) = 65°
∴ m (∠ BAC) = 180° – (65° + 65°) = 50° (The req.)

4

[a] ∵ E is the midpoint of $\overline{BC}$
∴ $\overline{ME} \perp \overline{BC}$ ∴ m (∠ MEA) = 90°
, ∵ $\overrightarrow{AD}$ is a tangent to the circle at D
∴ $\overline{MD} \perp \overrightarrow{AD}$ ∴ m (∠ MDA) = 90°
∴ From the quadrilateral ADME :
m (∠ DME) = 360° – (90° + 90° + 70°) = 110°
(The req.)

[b] ∵ $\overline{BC}$ is a diameter
∴ m (∠ BAC) = 90°
∴ m (∠ BAE) + m (∠ EDB) = 90° + 90° = 180°
∴ ABDE is a cyclic quadrilateral. (Q.E.D. 1)
∴ m (∠ CED) = m (∠ ABC)
, ∵ m (∠ ABC) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$
∴ m (∠ CED) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$ (Q.E.D. 2)

5

[a] m (∠ BAC) = $\frac{1}{2}$ m (∠ BMC) = $\frac{1}{2}$ × 120° = 60°
(inscribed and central angles subtended by $\overset{\frown}{BC}$)
(First req.)
, ∵ ABDC is a cyclic quadrilateral.
∴ m (∠ BDC) = 180° – 60° = 120° (Second req.)

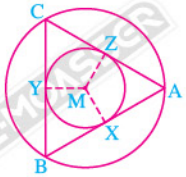
[b] ∵ m (∠ A) = $\frac{1}{2}$ [m $(\overset{\frown}{CE})$ – m $(\overset{\frown}{BD})$]
∴ 30° = $\frac{1}{2}$ [100° – m $(\overset{\frown}{BD})$]
∴ 60° = 100° – m $(\overset{\frown}{BD})$
∴ m $(\overset{\frown}{BD})$ = 100° – 60° = 40° (The req.)

10 Red Sea

1 1 b 2 a 3 d
4 c 5 c 6 b

2

[a] **Const. :** Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$
Proof : ∵ $\overline{AB}$, $\overline{BC}$, $\overline{AC}$
are tangent-segments to
the smaller circle at X , Y , Z
∴ $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{BC}$, $\overline{MZ} \perp \overline{AC}$
, ∵ MX = MY = MZ (radii lengths of the smaller circle)
∴ AB = BC = AC
∴ Δ ABC is an equilateral triangle. (Q.E.D.)



[b] m (∠ E) = m (∠ C) = 20°
(two inscribed angles subtended by $\overset{\frown}{BD}$) (First req.)
∵ ∠ CDE is an exterior angle of Δ ACD
∴ m (∠ CDE) = 43° + 20° = 63°
∴ m $(\overset{\frown}{CE})$ = 2 m (∠ CDE) = 2 × 63° = 126°
(Second req.)

3

[a] ∵ E is the midpoint of $\overline{BC}$
∴ $\overline{ME} \perp \overline{BC}$ ∴ m (∠ MEA) = 90°
, ∵ $\overrightarrow{AD}$ is a tangent to the circle at D

$\therefore \overline{MD} \perp \overrightarrow{AD}$ $\therefore m(\angle MDA) = 90°$

$\therefore$ From the quadrilateral ADME :

$m(\angle M) = 360° - (90° + 90° + 56°) = 124°$ (The req.)

[b] $m(\angle B) = \frac{1}{2} m(\angle M) = \frac{1}{2} \times 60° = 30°$

(inscribed and central angles subtended by $\overset{\frown}{AC}$)

(First req.)

, $\because \overline{MC} // \overline{BA}$, $\overleftrightarrow{MA}$ is a transversal

$\therefore m(\angle A) = m(\angle M) = 60°$ (alternate angles)

$\therefore$ In Δ BEA : $m(\angle BEA) = 180° - (30° + 60°) = 90°$

$\therefore \overline{BC} \perp \overline{AM}$ (Second req.)

4

[a] $\because \overline{XY} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

, $\because m(\angle C)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] In Δ ABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30°$

$\therefore m(\angle BAD) = 180° - 2 \times 30° = 120°$

$\therefore m(\angle BAD) = m(\angle DCE)$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180° - 120° = 60°$ (First req.)

, $\because \overline{AB}$ is a diameter

$\therefore m(\angle ADB) = 90°$

$\therefore$ In Δ ADB : $m(\angle ABD) = 180° - (90° + 60°) = 30°$

(Second req.)

[b] $\because m(\angle BCD) = \frac{1}{2} m(\angle M) = \frac{1}{2} \times 130° = 65°$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

, $\because \overline{AB} // \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore m(\angle ABC) = m(\angle BCD) = 65°$ (alternate angles)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC : $(\angle ACB) = m(\angle ABC) = 65°$

$\therefore m(\angle A) = 180° - 2 \times 65° = 50°$ (The req.)